SYMBOLS

S.I units are used in the K.C.S.E four year course including the symbols $+, -, \times, \div, \pm$ could also be used. The following rational symbols are important both in mathematics and physics:

$=$ Is equal to

$\neq$ Is not equal to

$>$ Is greater than

$\geq$ Is greater than or equal to

$<$ Is less than

$\leq$ Is less than or equal to

a: b Ratio of a to b

$\propto$ Varies as

$\equiv$ Is congruent to or identical to

$\simeq$ Approximately equal to

$\Leftrightarrow$ Is equivalent

$\Rightarrow$ Implies

$\therefore$ Therefore

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2. 00 NUMBERS

2.01 INTEGERS

Addition and Subtraction Rules:
1. Same signs: add and keep the sign (Same sign + same sign = same sign sum)
$3+5 = 8$; $-12 + -9 = -21$

2. Different signs: Subtract and keep the sign of the larger absolute number (Different sign + different sign = subtract, and keep the sign of the largest absolute value). The absolute value of a whole number is the same number but always positive.

$-45 + 32 = -(45 - 32) = -13$

REMEMBER: Subtraction is same as "Adding the Opposite." a – b = a + (-b)

To subtract numbers:
1. Change subtraction sign to addition.
2. Change the sign of the second number.
3. Follow rules for addition.

$37 - 45 = 37 + -45 = -(45 - 37) = -8$

Multiplication and Division Rules

1. Multiplying or dividing numbers with the same signs gives a positive answer.

$4\times 12 = 48$; $32 \div 2 = 16$; $-12 \div -4 = 3$; $-5 \times -6 = 30$

2. Multiplying or dividing numbers with different signs gives a negative answer

$4\times -12 = -48$; $32 \div -2 = -16$

3. Definition of division: $\frac{a}{b} = a \times \frac{1}{b}$

$\frac{12}{5} = 12\times \frac{1}{5}$

When several operations are applied on integers please use**: BEDMAS.**

(B) BRACKETS (E) EXPONENT (D) DIVISION (M) MULTIPLICATION or of (A) ADDITION (S) SUBTRACTION – B.E.D.M.A.S

$\frac{-2(5+3)-9\div 3+5}{-3\times -5+-2\times 4}$ 2010 p1 no 1. $\frac{-2(8)-3+5}{15+(-8)}\sqrt{1}\ \textit{for brackets multiplication and division}$

$\frac{-16+2}{7}\sqrt{1}\ \textit{for addition and division} = -2\sqrt{1}\ \textit{for answer}$

2.02 Prime numbers /Factors

These are numbers having exactly two factors: one and itself. These include 2, 3, 5, 7, 11, 13, 17, 19, 23 etc. All composite numbers can be written as products of prime factors using a factor ladder or a factor tree. Begin with the least prime number that is a factor. Repeat until the quotient is prime or one.

Express 10500 in terms of prime factors. 2011 p1 no 14.

10500	
2	5250
2	2625
3	875
5	175
5	35
5	7
7	1

10500 = 2^2 x 3 x 5^3 x 7 $\sqrt{1}$ *for answer.*

2.03 G.C.D OR H.C.F /LCM

Writing numbers using prime factors can help in getting the GCD and LCM of numbers. The **L.C.M – LEAST COMMON MULTIPLE-** Is the smallest number that a set of given numbers divides evenly into. The L.C.M is useful in finding a common denominator when adding or subtracting fractions.

TO FIND THE L.C.M OF A SET OF NUMBERS:

Write the set of numbers one after the other on the same line. Use the ladder factor method to get the prime factors of the numbers.

Start with the common prime factors only (these can be used to give you the GCD). Only common factors can be multiplied to give the G.C.D.

Then finish up with the remaining prime factors.

The L.C.M is the product of ALL the common prime factors and the remaining prime factors.

(G.C.D = product of common prime factors only)

The prime factors are very useful tools in getting squares and cube roots of numbers.

For example: Three bells ring at intervals of 9mins, 15mins and 21 mins. The bells ring together at 11.00 pm. Find the time the bells will ring together again.

	9	15	21
3	3	5	7
3	1	5	7
5	1	1	7
7	1	1	1

$\sqrt{1}$ *for table or alternative*

Bells ring after 3^2x5x7 = 315 minutes =5hrs 15mins$\sqrt{1}$ *for numbers of hours and mins*

Bells ring at 4.15a.m$\sqrt{1}$ *for correct time*

2.04 FRACTIONS AND DECIMALS

Proper fraction – A fraction such that, little number on top (numerator); big number on bottom (denominator) $\boldsymbol{e.g}\ \frac{12}{17}$

Improper fraction – A fraction with a value greater than or equal to 1 e.g $\frac{45}{7}$

Mixed number - A whole number and a fraction written together $\boldsymbol{e.g}\ 34\frac{6}{7}$

To express a mixed number as an improper fraction, first multiply the whole number by the denominator and add the numerator. Then write this sum over the denominator. $\boldsymbol{e.g}\ \mathbf{6\frac{4}{5}} = \frac{\mathbf{5\times6+4}}{\mathbf{5}} = \frac{\mathbf{34}}{\mathbf{5}}$

To express an improper fraction as a mixed number, divide the denominator into the numerator**.** $\boldsymbol{e.g}\ \frac{\mathbf{34}}{\mathbf{7}} = \mathbf{4\frac{6}{7}}$

To add or subtract a set of fractions, you must make sure they have a Common Denominator which will be the L.C.M of the set of denominators. With the L.C.M rewrite the fractions as equivalent fractions and then add or subtract fractions by adding or subtracting numerators together and reduce answer to lowest terms.

$$\mathbf{\frac{5}{7}+\frac{2}{5}-\frac{3}{6}=\frac{150}{210}+\frac{84}{210}-\frac{105}{210}=\frac{150+84-105}{210}=\frac{129}{210}=\frac{43}{70}}$$

To add or subtract mixed fractions, Whole numbers are added together first. Then determine LCM for fractions. Reduce fractions to their LCM. Add numerators together and reduce answer to lowest terms. Add sum of fractions to the sum of whole numbers.

$5\frac{1}{3}+2\frac{1}{5}-3\frac{1}{2} = (5+2-3)+\left(\frac{1}{3}+\frac{1}{5}-\frac{1}{2}\right) = (4)+\left(\frac{10}{30}+\frac{6}{30}-\frac{15}{30}\right) = 4+\frac{1}{30} = 4\frac{1}{30}$

To multiply fractions change any mixed fractions to improper fractions before multiplying. Then multiply numerator to numerator and denominator to denominator.

$\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$

To divide by a fraction, multiply by its inverse or reciprocal.

$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}$

Mixed fractions should be changed into improper fractions first before dividing or multiplying them. To divide fractions, multiply the first fraction with the reciprocal of the second fraction.

When several operations are applied on fractions please use**: BEDMAS.**

2.05 SQUARES, SQUARE ROOTS

Some K.C.S.E questions could require simplification using the rules on integers and fractions.

To solve these types of problems change the numbers involved into products of prime factors. If decimals are involved, write them as suitable whole numbers that are multiples of powers of ten first. Remember $\sqrt[n]{a} = a^{\frac{1}{n}}$. Then use the factor method to quickly sort out the questions with great accuracy. In almost all the questions in this area do not require the use of a four figure table or a calculator. A calculator and four figure mathematical tables MUST not be used.

2.06 CUBES AND CUBE ROOTS

The cube root of a number can be found using factor method, the four figure table of the calculator.

1. To use factor method, the number is written as a product of prime factors as indicated above for square roots.

2. To use a four figure table, some numbers must be written as suitable powers of ten before using the tables to find the answer.

2.07 RECIPROCALS

The reciprocals section of the four figure table can be used to find the reciprocals of numbers. Some numbers must be suitably written as powers of ten first before checking for their reciprocals. Do not forget to correctly use the reciprocal of the power of ten.

2.08 RATIOS, RATES, PERCENTAGES AND PROPOTIONS

To divide a number in a stated ratio, first find the total ratio and then multiply the number by $\frac{ratio}{total\ ration}$ for each ratio or as is required.

Percent increase describes an amount that has grown and **percent decrease** describes an amount that has reduced.

Percent of Change = $\frac{\boldsymbol{amount\ of\ change}}{\boldsymbol{original\ amount}}$ or $\frac{amount\ of\ increase\ or\ decrease}{original\ amount}$, expressed as a percent.

To change a decimal to a %, move decimal point two places to right and write percent sign.

2.09 COMPOUND PROPORTIONS

Combining ratios a:**b** and **c**:d , the numbers in the bold positions must have the same value for the ratio to combine and become the ratio x:y:z. Hence, multiply the ratio as

c (a: b) and **b**(c: d) to give ac: bc and bc: bd. **So x:y:z = ac: bc : bd**

Remember: a: b = x: y means x/y = a/b

Compound proportions questions related to work could be sorted out a table as shown below:

quantity name	Q1	Q2	Q3
Value 1	a	b	c
value 2	d		e

To find the missing number for Q2, check on how the variations in Q1 and Q3 will affect b, in terms of whether the ratio will be an increasing one or a decreasing one and multiplying accordingly.

For example :1993 p2 no 5 It takes 30 workers 6 days working 8 hours a day to harvest maize in a farm. How many days would 50 workers working 6 hours a day take to harvest the maize?

quantity	workers	days	hours
value 1	30	6	8
value 2	50	x	6

Increasing workers reduces days of working $\rightarrow \frac{30}{50}$, reducing working hours increases days of working $\rightarrow \frac{8}{6}$, hence x = $6\times\frac{30}{50}\times\frac{8}{6} = 4\frac{4}{5}$ days .

2.10 RATES OF WORK

If B can complete work in b hours, then B does work at the rate of $\frac{1}{b}$ per hour. If C can complete the work in c hours, the rate of work is $\frac{1}{c}$ per hour. If B and C work together the rate of work will be the sum of their rates ($\frac{1}{b}+\frac{1}{c}$) per hour . Time to complete the work when working together will be the reciprocal of the Sum.

For Taps, the rates of the taps bringing in water are summed up. If a drain pipe or tap is involved, the difference of the rates of the drain tap to the filling taps gives the rate at which the tank accumulates water. The time to fill the tank equals the reciprocal of the difference.

You may be given conditions in a question that may require you to use rates step by step.

If x/y of the work is remaining and B working at the rate of a/b is to complete this work, B will take ($\frac{x}{y} \div \frac{a}{b}$) of time.

If the rate of working is $\frac{a}{b}$ of time and work is done at this rate for x duration of time, then $\left(\frac{a}{b}\right)(x)$ of work has been done in that time. And $\left[1 - \left(\frac{a}{b}\right)(x)\right]$ of work remains.

Time taken to complete any work = fraction of work ÷ rate of working. Units of time should be used consistently.

2.11 MIXTURES

Proportions as stated above apply in mixtures.

1. If A, B, and C are mixed in the ratio a:b:c, then a units of A, b units of B and c units of C are mixed together. If the costs per unit of A, B and C are x, y and z respectively, the cost price or buying price per unit of the mixture will be**:** ($\boldsymbol{\frac{ax+by+cz}{a+b+c}}$ **).**

2. $(percentage\ profit\ \times cost\ price\ or\ buyinh\ price) = (profit)$

3. If A and B are at cost prices a and b , the mixture will be cost priced at x if and only if the ratio of the mixture of A:B is: (difference between **b** and **x**): (difference between **a** and **x**) in its simplest form. This x is the buying price of the mixture. If profit is to be made, then the selling price must be determined normally.

2.12 LINEAR MOTION

1. Speed $= \frac{distance}{time\ taken}$ 2. Acceleration $= \frac{change\ in\ velocity}{time\ taken} = \frac{final\ velocity - intial\ velocity}{time\ taken}$

3. Relative speed = sum of speeds if the two bodies are approaching each other.

4. Relative speed = difference of speeds if two bodies are moving in the same direction as to overtake each other.

5. To calculate the time taken for objects to meet or overtake each other, the distance between the objects is used:

Time taken $= \frac{Distance\ between\ the\ objects}{relative\ speed}$

6. Word problems in motion could require the formation of linear equations to be solved.

7. Motion graphs:

- The gradient of a distance – time graph is equal to the speed of the object.
- The gradient of a speed – time graph is equal to the acceleration of the object.
- The area under a velocity – time graph is equal to the distance moved.
- The gradient of a straight line joining two points on a motion graph gives the average rate of change.
- The gradient of a tangent at a point gives the instantaneous rate of change at that point.

2.13 INDICES, LOGARITHMS AND FURTHER LOGARITHMS

INDICES

1. $a^m \times a^n = a^{m+n}$; 2. $a^m \div a^n = a^{m-n}$; 3. $a^0 = 1$; 4. $a^{-m} = \frac{1}{a^m}$; 5. $a^{\frac{1}{n}} = \sqrt[n]{a}$; 6. $\sqrt[n]{a^m} = a^{\frac{m}{n}}$ 7. $(a^m)^n = a^{mn}$ 8. $a^1 = a$.

2.14 LOGARITHMS USING MATHEMATICAL TABLES

For all v, w>0;

1.$\log(vw) = \log v + \log w$ 2.$\log(\frac{v}{w}) = \log v - \log w$

3.$\log(v)^k$ = k$\log v$ 4. log 1 = 0

5. $\log_k k = 1$ 6. Change of base, log_ka = $(\frac{\log_c a}{\log_c k})$

7. $\log_a c = \frac{1}{\log_c a}$ 8. $\log_a \sqrt[n]{x} = \frac{1}{n}\log_a x$

If n = m x 10^c where 1≤m<10, then $\log n = \log m + c\log 10$ = c + $\log m$, c is the characteristic of n. $\log m$ is the mantisaa of n gotten from the four figure mathematical tables.

Hence $\log n$ = mantissa + characteristic.

To use the logarithms four figure table:

1. Write the number in standard form: n = m x 10^c

2. Identify the first four numbers of m say – abcd

3. Select row 'ab' from the logs tables ; 4. locate number at column 'c' from the row 'ab' say x.

5. If d≠0, locate number at column 'd' of mean difference from the row 'ab' say y. (if d = 0 , then y=0)

$\log n$ = c + $.(x+y) = c.(x+y)$

Antilog can be found using the following procedure:

1. If $n = m.abcd$, and n≥0, the Antilog (n) = $.(x = y) \times 10^{(m+1)}$;

If $n = \bar{m}.abcd$, and n<0, the Antilog (n) = $.(x+y)$x $10^{(-m+1)}$

2. Select row 'ab' from the antilog table.

3. Select column 'c' of row 'ab' from the antilog table to locate the number say x.

4. Select the column 'd' of mean difference of row 'ab' from the antilog table to locate the number say y.

(if d=0 then y=0). If $\bar{4}$ + 0.0867 = $\bar{4}$.0867

NOTE:

1. Logarithms add up if their numbers multiplied. logarithms subtract if their numbers divided. for these two the normal rules of addition and subtraction applies.

2. A logarithm of the form $m.abcd$, can be divided by a whole numbers like any decimal. Extra care should be taken for a logarithm of the form ($\bar{m}.abcd$).

For $\frac{\bar{m}.abcd}{n}$, express $\bar{m}.abcd$ as ($\bar{m}$ + $.abcd$)$/n$. If m is exactly divisible by n proceed with division normally, if not make the following adjustments. Go for the next number greater than m that can be divided exactly by n say p. Then rewrite the expression as $\frac{\bar{p}+(p-m).abcd}{n}$ and proceed with division normally. For example $\frac{\bar{4}\ .5678}{3}$, becomes $\frac{\bar{6}+(6-4).5678}{3}$ = $\frac{\bar{6}+2.5678}{3}$ = $\bar{2}$ + .8559 = $\bar{2}$.8559

2.15 FURTHER LOGARITHMS

If log_ay = x , then y = a^x which can be used to solve logarithmic equations.

Laws of logarithms are used to simplify the equations that lead to their solutions. Normal laws of solving equations also apply.

1. log_ax + log_ay = log_axy   2. log_ax - log_a y = $log_a\left(\frac{x}{y}\right)$ 3. xlog_ay = $log_a(y^x)$

4. $log_a(a)$ = 1   5. log_a1 = 0

Resulting equations are solved using the normal laws.

For equations of the form c(a^{2x}) + b(a^x) + d = 0, let y = a^x and reduce the equation to a quadratic equation, i.e cy² + by + d = 0.

2.16 APPROXIMATIONS AND ERRORS

- True value – estimated value = error
- absolute error = $\frac{1}{2}$ $(smallest\ possible\ value\ with\ the\ instrument)$
- Relative error r. e = $\frac{error}{True\ value\ or\ actual\ measured\ value}$
- Percentage error p. e = $\frac{error}{True\ value}$ x 100
- When numbers add or subtract, the absolute errors of each value are summed up.
- When numbers multiply or divide, their percentage or relative errors are summed up.
- The absolute error is = $\left(\frac{1}{2}\right)(the\ value\ of\ the\ smallest\ division\ on\ the\ scale.)$
- If a number p, is raised to power n i. e $(p)^n$ the percentage error or relative error in$(p)^n$ will be $(n)(relative\ error\ in\ p)$

3.00 MEASUREMENTS I

3.01 LENGTH AND PERIMETERS

1. You should know how to change mm ↔ cm ↔ m ↔ km

2. Perimeter is total length all round the plane figure.

3. Arc length = $\frac{\theta}{360}$ x2πr (angle θ and radius r)

3.02 PYTHAGORUS THEOREM

"For any right triangle, the sum of the areas of the two small squares is equal to the area of the larger square."

$a^2 + b^2 = c^2$.

3.03. AREAS OF AND VOLUMES OF SIMPLE PRISMS

You should know how to change $cm^2 \leftrightarrow m^2 \leftrightarrow km^2$

1. Surface area of a closed cylinder = $2\pi r$ (r + h); if open on one end A = $\pi r^2 + 2\pi rh$ (h = height of cylinder and r = radius)

2. Surface area of a prism = (2 x area of cross – section) + (perimeter of cross- section x length)

3. Volume of a cube = l^3 $(where\ l\ is\ the\ length)$

4. Volume of a cuboid = $l.b.h$ ($l, b\ and\ h$ being its lengths)

5. Volume of a prism = area of cross-section (A) x length (l) = (Axl)

6. Volume of a cylinder = πr^2 h (radius r, and height h)

3.04 VOLUME- CAPACITY RELATIONSHIP

Capacity is the amount a container can hold in litres.

1 litre = $1000cm^3$; $1cm^3$ = 1 millitre; $1m^3$ = 1000 litres.

3.05 MASS, DENSITY AND WEIGHT

Weight (W) = mass(m) x gravity(g) or W = mg.

Density = $\frac{mass}{volume}$ in g/cm^3 or kg/m^3.

3.06 TIME:

1) 12 HRS TO 24 HRS AND VICE VERSA

To change 12 hours time to 24 hours time, only add 12 to the hours between 1.00pm to 11.00pm. (Add 12 to the pm hours only.) Mid-night becomes 0000hrs.

To covert 24 hours to 12 hours time, take away 12 from the hours between 1300hrs to 2300hrs. (0000hrs becomes 12.00 mi-night)

To calculate time intervals in minutes from one hour to the next:

- Count the minutes to the next hour
- Count the hours to the finish hour
- Add on the first minutes to the finish minutes
  If the time interval of minutes adds up to more than 60mins, change this time to hours and minutes
- Total the hours and the minutes to get the total time interval.

2) TRAVEL TIME TABLES

These tables give the destination, expected time of arrival and departure for vehicles, ships, trains and airplanes. Time intervals can be calculated from here.

4.00 MEASUREMENTS II

4.10 FURTHER AREAS

4.11 AREA OF TRIANGLES

1. Area of a triangle = $\frac{1}{2}$ x base (b) x height(h) = $\frac{1}{2}$ bh = $\frac{1}{2}$ ab sinΘ where Θ is the included angle for lengths a and b.

2. Area of triangle with sides a, b, and c = $\sqrt{s(s-a)(s-b)(s-c)}$ where s =$\frac{a+b+c}{2}$

3. Area of triangle that is part of a sector = $\frac{1}{2}r^2$ sinΘ. (r=radius of circle)

4.12 AREA OF QUADRILATERALS

1. Area of a parallelogram = base (b) x height(h) = bh = ab sinΘ, where Θ (if lengths a and b are given and the included angle Θ)

2. Area of a trapezium = $\frac{1}{2}$ (sum of parallel side lengths) x height = $\frac{1}{2}$ (a+b)h

= $\frac{1}{2}$ (a + b) x c sinΘ where a and b are the parallel lengths, c the third side and Θ is the included angle.

3. Area of a kite and rhombus = $\frac{1}{2}$(product of diagonal length)

4.13 AREA OF PARTY OF A CIRCLE

SECTOR

1. Area of a circle = πr^2 (r=radius)

2. Area of a sector = $\frac{\theta}{360}\pi r^2$

4.14 SEGEMENT

1. Area of a segment = area of sector – area of triangle = $\frac{\theta}{360}\pi r^2$ - $\frac{1}{2}r^2$ sinΘ.

4.15 AREA OF A REGULAR POLYGON

Area of a regular polygon with n sides of each length a and interior angle Θ

A = $\frac{n}{4}a^2 \tan\frac{1}{2}\theta$

If the side length is a, and number of sides n the area A = $\frac{a^2 n}{4\tan\left(\frac{180}{n}\right)}$

4.16 COMMON REGIONS BETWEEN TWO CIRCLES

Areas of composite figures can be found by finding the area of each shape and either adding or subtracting them as the diagram dictates. To find the common area between intersecting circles, find the area of each segment and sum up.

4.20 SURFACE AREA OF SOLIDS

4.21 PRISM

Surface area of a prism = (2 x area of cross – section) + (perimeter of cross- section x length)

4.22 CONE

Curved surface area of a cone = πrl. Total surface area = πr(l + r)

4.23 PYRAMIDS

Surface area of a pyramid = (area of polygon base) + (area of its triangular faces) = B + $\frac{1}{2}$ PL where

B = base area; P = base perimeter and L = length perpendicular to side of base.

4.24 FRUSTUM

1. Surface area of a frustum = area of the top and bottom surfaces + area of its other slanting surfaces.

2. Surface area of a frustum cone of radii r and R, slant height L and vertical height h:

Curved surface area = πL (R + r) where L = $\sqrt{h^2 + (R - r)^2}$

Total surface area = πL(R+r) + π(R^2 + r^2)

4.25 SPHERES

Surface area of a sphere = $4\pi r^2$

4.26 HEMISPHERES

Curved surface area = $2\pi r^2$.

Surface area of a solid sphere = $3\pi r^2$.

4.27 SURFACE AREAS OF COMBINED SOLIDS

Splitting these complicated solids into simpler solids will help in finding their surface areas.

4.30VOLUME OF SOLIDS

4.31 PRISM

Volume of a prism = area of cross-section x length. ($A \times l$)

4.32 PYRAMID

Volume of a pyramid = $\frac{1}{3}$ (base area) x height = $\frac{1}{3}$ Ah.

4.33 CONE

For a cone Volume = $\frac{1}{3}\pi r^2 h$

4.34 FRUSTUM

1. Volume of frustum of a cone = $\frac{1}{3}\pi$h(R^2 + r^2 + Rr), h is vertical height.

2. Volume of a frustum of a pyramid = Volume of bigger pyramid – Volume of smaller pyramid.

4.35 SPHERE

Volume of a sphere = $\frac{4}{3}\pi r^3$

4.36 HEMISPHERE

Volume of a solid hemisphere is half that of the sphere = $\frac{2}{3}\pi r^3$

4.37 VOLUMES OF COMBINED SOLIDS

These can be found by splitting the solids into simpler solids whose volumes you can easily find.

N.B: FROM ONE SOLID TO ANOTHER

In manufacturing process a solid could be made into a different solid or solids. If there is no wastage, the volume of the solids before and after processing is the same. (can be equated)

5.00 ALGEBRA

5.10 ALGEBRAIC EXPRESSIONS

5.11 SIMPLIFICATION OF ALGEBRAIC EXPRESSIONS

In like terms, the letters and powers of each letter must be the same.

Only like terms can be **added** or **subtracted** to give a single term.

To multiply or divide expressions, multiply or divide the numbers first. Then use the basic laws of indices to simplify sets of the same letters.

5.12 REMOVAL OF BRACKETS

Multiply the term outside the bracket by each of the terms inside the brackets. When there is a negative sign in front of the brackets, the signs that were inside the brackets change when the brackets are removed.

5.13 FACTORIZATION BY GROUPING

When grouping terms be careful with the signs. Expressions containing four terms can be factorized by grouping the four terms into two pairs so that each pair has a common factor. Factor out the terms for each pair, and complete the factorization.

5.14 SUBSTITUTION AND EVALUATION

Substitution is replacing letters by numbers. This is used to find the numerical value for the expression. Make proper use of brackets if negative numbers are involved.

5.20 LINEAR EQUATIONS

5.21 LINEAR EQUATIONS IN ONE UNKNOWN

Any equation that can be written in the form $Ax + B = C$, where A, B, C are real numbers is a linear equation.

Some equations may require simplification before solving them. Brackets may require removal then gather like terms to one side and the known numbers to one side. Simplify both sides and solve.

It could be necessary to find the L.C.M of algebraic denominators, then rewrite the expressions and simplify them, then follow the steps indicated above to solve the linear equation that arises.

5.22 LINEAR EQUATIONS FROM REAL LIFE SITUATIONS

Statements in words are often written as algebraic expressions in mathematics. Any letter may be used to stand for the unknown number, but a different letter must be used for each different unknown.

For statements in words just copy what is stated in symbols (each symbol used should represent what is unknown) to form the equation to be solved. The following steps could help:

1. Read the problem and assign symbols to the quantities in the problem.

2. Form an equation using the symbols from Step 1. Include a symbol for the unknown quantity (usually x, but other symbols are also used).

3. Solve the equation from Step 2 for the unknown quantity.

'Letter terms' = 'number terms'

Of course, the only way to learn to solve word problems is through practice. This is calisthenics for the brain

5.23 SIMULTANEOUS EQUATIONS

To solve simultaneous equations use elimination method unless stated otherwise.

a) ELIMINATION METHOD

Step 1: Put the equations in Standard Form first (ax + by = c)

Step 2: Inspect the equations. **Same number of x's or y's?.** If the sign is different, ADD the equations, otherwise subtract them to get one equation in with one unknown, solve this equation. Substitute the answer into one of the original equations to get the other unknown. CHECK by substitution of BOTH answers.

If the coefficients are not the same, you can make one of the unknowns have equal coefficients by multiplying all the terms in either one or both equations by appropriate numbers. Then proceed as above.

b) SUBSTITUTION METHOD

To solve simultaneous equations by substitution:

1. Make one unknown the subject of one equation

2. Substitute this expression for the unknown in the other equation

3. Solve the resulting equation to find one unknown.

4. Find the other unknown by substituting your solution in any of the original equation.

5.30 EQUATIONS OF STRAIGHT LINES

5.31 GRADIENT OF A STRAIGHT LINE

If two point on a line are known as (a,b) and (c,d) then the gradient m = $\frac{d-b}{c-a} = \frac{difference\ of\ lasts}{difference\ of\ firsts}$ correspondingly.

5.32 EQUATION OF A STRAIGHT LINE

1. The slope- intercept form of Equations of straight lines is y = mx + c, where m is the gradient and c the y-intercept.

2. The general form of the equation of a straight line is or ax + by = c or ax +by +c = 0

3. Horizontal Lines equation are of the form y = k.

4. Vertical lines equations are of the form x = k

5. If a line passes through the point (a, b) and has gradient m, then its equation is

y – b = m (x – a), which can be reduced to any other suitable form.

6. The equation of a line passing through the points with coordinates $(a, b)\ and\ (c, d)$ is

$$\frac{y-b}{d-b} = \frac{x-a}{c-a}$$

ALTERNATIVELY: For the equation of a line through two known points (a, b) and (c, d), find the gradient m as in (1), and then state the equation as in (2)

7. The length between two points (a,b) and (c,d) is given by $\sqrt{(d-b)^2+(c-a)^2}$

5.33 PERPENDICULAR LINES AND THEIR GRADIENTS

If two lines, $y = m_1x + c_1$ and $y = m_2x + c_2$ are perpendicular, the product of their gradients is – 1; $m_1 \times m_2 = -1$

If the lines $a_1x + b_1y + c_1$ and $a_2x + b_2y + c_2$ are perpendicular then $a_1a_2 + b_1b_2=0$

Perpendicular lines have negative reciprocal gradients, so if you need the gradient of a line perpendicular to a given line, simply find the gradient of the given line, take its reciprocal (flip it over) and MULTIPLY by -1.

5.34 PARALLEL LNES AND THEIR GRADIENTS

If two lines are parallel, their gradients are equal; $m_1 = m_2$

If the lines $a_1x + b_1y + c_1$ and $a_2x + b_2y + c_2$ are perpendicular then $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

5.35 THE X – AND Y – INTERCEPTS OF A LINE

1. The line $(ax + by + c = 0)$ meets the x – axis at the point $\left(-\frac{c}{a}, 0\right)$, meets the y-axis at the point $\left(0, -\frac{c}{b}\right)$ and has gradient$\left(-\frac{a}{b}\right)$.

2. If a and b are the x – intercept and y – intercept respectively, the equation of the line is given as

$\frac{x}{a} + \frac{y}{b} = 1$

Lines meet the x – axis when y = 0; and the y – axis when x = 0. Equations that result can be solved.

3. Rem. m = tanΘ, where Θ is the angle the line makes with the horizontal or even the x – axis.

4. When two lines are intersecting, to find the point of intersection solve the simultaneous equations that make the lines.

5.40 QUADRATIC EXPRESSIONS AND EQUATIONS I

5.41 EXPAND ALGEBRAIC EXPRESSIONS

To find the product of two brackets, multiply each term in one bracket by each tern in the other bracket; OR To multiply two brackets use F.O.I.L i.e Products of **Firsts** + **O**uter + **I**nner + **L**asts and then Simplify.

5.42 QUADRATIC IDENTITIES

Rem. $(a + b)(a + b) = a^2 + 2ab + b^2$; $(a - b)(a + b) = a^2 - b^2$;

$(a - b)(a - b) = a^2 - 2ab + b^2$. These expressions can be used directly to factorize expressions especially the difference of two squares quadratic expressions.

5.43 FACTORIZING QUADRATIC EXPRESSIONS

1. To factorize an expression with four terms, group terms into two pairs each with common factors. Then factorize accordingly.

2. To factorize an expression with three terms like $ax^2 + bx + c$, find the factors of (ac) which add up to (b). Then rewrite bx in terms of these factors. The statement is now made of four terms and grouping can be used to factorize it.

3. When requested to simplify algebraic expressions, factorization is mostly applied. When factorizing any expression first look at what is common between all the terms first before you proceed.

5.44 SOLVING QUADRATIC EQUATIONS BY FACTORIZATION

$ax^2 + bx + c = 0$ can be solved as follows:

- If b=0, i.e $ax^2 + c = 0$, isolate x^2 directly and square-root both sides to get the values of x.
- If c = 0, i.e $ax^2 + bx = 0$, factor out the common term immediately. Use the null factor law to solve for x.
- If a, b , c are non-zero, see whether you can easily factorize the equation using factors that product ac and sum b. Use the null factor law to solve for x.

5.50 QUADRATIC EXPRESSIONS AND EQUATIONS II

5.51 FURTHER FACTORIZATION OF QUADRATIC EXPRESSIONS

To factorize an expression with three terms like, $ax^2 + bxy + cy^2$, find the factors of (acx^2y^2) that add up to (bxy). Then rewrite the expression with the middle term split into two terms using these factors. They are now four terms and the expressions can be factorized by grouping. For example : $3x^2 - 4xy + y^2$, the prod. is $3x^2y^2$ and sum is $-4xy$, factors are $-xy$, and $-3xy$.

Hence $3x^2 - xy - 3xy + y^2 = x(3x - y) - y(3x - y) = (x-y)(3x-y)$

5.52 COMPLETING THE SQUARE

$x^2 + (bx) = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$ is a completed square.

When the coefficient of x^2 is not one, we start by taking out the coefficient of x^2 from the terms in x. We then complete the square for the expression in brackets.

$ax^2 + bx + c$ becomes $\left(a(x^2 + \frac{bx}{a}\right) + c$; hence $(a)\left[\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right] + c$,

OR $ax^2 + (bx) + c$ becomes $(a)\left[\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b^2-4ac}{4a^2}\right)\right]$ which is a complete square which can be used to solve the quadratic equation whose coefficients are a, b, and c.

$That\ is\ ax^2 + (bx) + c = 0 = (a)\left[\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b^2-4ac}{4a^2}\right)\right] = 0$ and x can be found.

5.53 QUADRATIC FORMULA

Quadratic formula: $x = \frac{-(b) \pm \sqrt{(b)^2 - 4(a)(c)}}{2(a)}$

- If $b^2 - 4ac > 0$, the two roots are real and different.
- If $b^2 - 4ac = 0$, the roots are repeated (equal roots)
- If $b^2 - 4ac < 0$, no real roots exist.

If an expression does not easily factorize or does not factorize or you have been asked in the question to estimate the answer, use the quadratic formula method to solve for x. (In fact if instructions to factorize do not appear in the question always use the quadratic formula to solve quadratic equations.)

5.54 TABLE OF VALUES FOR A GIVEN QUADRATIC RELATION

A table of values to be completed is normally given. Use your calculator carefully, especially when dealing with negative values, decimals and fractions to complete the table. Make proper use of brackets.

5.55 GRAPHS OF QUADRATIC EQUATIONS AND SOLUTIONS

Write down the (x, y) coordinates of the points from your table of values. Choose a suitable scale or use the given scale to plot the points with the coordinates you have listed. Mark the points with a cross or circled dot. Use your free hand to draw the curve through the points. Label the curve with its equation.

To graph a linear equation, find the (x, y) values for three points only. Two of these points give you the line. The third acts as a check.

5.56 SIMULTANEOUS EQUATIONS, ONE LINEAR AND THE OTHER QUADRATIC

These are solved normally by substitution. One unknown in the linear equation is made the subject of the linear equation.

Then substitute this expression into the quadratic equation. This gives a quadratic equation in one unknown, which can be solved.

Then substitute each of the solutions in turn into the linear equation to find the values of the other unknown. Match the answers properly and correctly.

5.60 LINEAR INEQUALITIES

5.61 INEQUALITIES ON A NUMBER LINE

1. To represent inequalities on a number line use an open circle  for an excluded point, i.e for signs $<$ and $>$.
Use filled circle  for included point, i.e for signs $\geq$ and $\leq$.

$<$ - less than 

$>$ - greater than 

$\geq$ - greater than or equal to 

$\leq$ - less than or equal to 

5.62 SIMPLE LINEAR INEQUALITIES IN ONE UNKNOWN

2. When solving linear inequalities in one unknown, all the properties on solving linear equations apply with the exception that: If you multiply or divide by a negative number, you must REVERSE the inequality sign.

5.63 COMPOUND INEQUALITY STATEMENTS IN ONE UNKNOWN

To solve compound inequalities in one unknown, split the inequality suitably into two simultaneous inequalities and proceed as in 2 above. Then combine the results.

5.64 GRAPHICAL REPRESNTATION OF LINEAR INEQUALITIES

To solve inequalities in two unknowns graphically draw the boundary line which is dotted if the signs $>$ or $<$ have been used; or the boundary line is continuous if the signs $\leq$ or $\geq$ have been used.

5.65 SOLUTIONS OF LINEAR INEQUALITIES

Choose an easy point on one side of the line, and find out whether its co-ordinates satisfy the inequality or not. Shade the unwanted side or region.

5.66 FORMATION OF SIMPLE LINEAR INEQUALITIES FROM GRAPHS

1. To form inequalities from inequality graphs, determine the equations of the lines first using the normal method of coordinates and gradients. OR importantly write the equations of each as the intercept form of a straight line, $\frac{x}{a}+\frac{y}{b}=1$ where a, and b are x- and y- intercepts respectively.

2. To fit the inequality signs choose a test point in the wanted region, substitute in the equation and fit in the inequality accordingly. Be careful with the issue of dotted and continuous lines.

5.70 LINEAR PROGRAMING – LINEAR INEQUALITIES II

5.71 FORMATION OF LINEAR INEQUALITIES

1. Important Words include:

At least - means greater than or equal to

At most –means less than or equal to

No more than - means less than or equal to

More than - means greater than

Less than - means less than

Between -means compound inequality with

Inclusive -means "make the or equal to line"

2. Formulating inequalities from word statements is an art that can only be mastered with practice and experience. Every Linear Programming problem has some unique features, but most problems also have common features.

5.72 GRAPHICAL SOLUTIONS OF LNEAR INEQUALITIES AND OPTIMIZATION

The linear inequalities identified appropriately are graphed and the unwanted region shaded for each inequality one at a time.

After the wanted region has been identified, an objective function or search line is obtained, equated to a suitable constant and drawn. This search line is then:

1. Moved parallel to itself away from the origin towards large values without entirely leaving the feasible region to maximize. Any feasible solution on the objective function line with the largest value is an optimal solution.

2. Moved parallel to itself towards the origin toward smaller **values** without entirely leaving the feasible region to minimize**.** Any feasible solution on the objective function line with the smallest value is an optimal solution.

6.00 SURDS

6.01 SIMPLIFICATION OF SURDS

A surd is an irrational root of a rational number. $\sqrt[n]{a}$ is a surd of order n if its root cannot be found exactly.

1. ADDITION AND SUBTRACTION OF SURDS:

Surds can only be added or subtracted if and only if they are of the same order and the number under root is the same.

Factor out the root and simplify the numbers in brackets. If what is under root is a composite number write it in its simplest form using prime factors and proceed as above.

Remember: $\sqrt{a \pm b} \neq \sqrt{a} \pm \sqrt{b}$

2. MULTIPLICATION: $\sqrt{ab} = \sqrt{a}\,\sqrt{b}$ and surds can be multiplied regardless.

6.02 RATIONALIZATION OF DENOMINATORS

1. MULTIPLICATION: $\sqrt{ab} = \sqrt{a}\,\sqrt{b}$ and surds can be multiplied regardless.

2. FOR DIVISION: SURDS ARE SIMPLIFIED VIA RATIONALIZING THE DENOMINATOR:

Remember: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$,

To simplify fractional surd into the form a$\sqrt{b}$ + c$\sqrt{d}$ the denominator is rationalized by multiplying the numerator and denominator by the conjugate of the denominator.

The conjugate of $\left[\left(n\sqrt{a} \pm m\sqrt{b}\right)\right]$ is$\left[\left(n\sqrt{a} \mp m\sqrt{b}\right)\right]$.

- If a fraction is in the form $\frac{1}{\sqrt{a}}$, then multiply both the top and bottom of the fraction by $\sqrt{a}$ to rationalize the denominator.
- If the fraction is in the form $\left(\frac{1}{a-\sqrt{b}}\right)$, then multiply both the top and the bottom of the fraction by $\left(a+\sqrt{b}\right)$ to rationalize the denominator.
- If the fraction is in the form $\left(\frac{1}{a+\sqrt{b}}\right)$, then multiply both the top and the bottom of the fraction by $\left(a-\sqrt{b}\right)$ to rationalize the denominator.

7.00 SEQUENCES AND SERIES

7.01 A.P SERIES

The n^{th} term of the arithmetic progression or series: $(a) + (a + d) + (a + 2d) + (a + 3d)$ is $\boldsymbol{T_n = [a + (n-1)d] = l}$

7.02 G.P SERIES

1. The n^{th} term of a geometric progression or series $(a) + (ar^1) + (ar^2) + (ar^3) +$…… is $\boldsymbol{T_n = ar^{n-1}}$.

7.03 SUM OF A.P SERIES

The sum of the A.P is $\boldsymbol{S_n = \frac{n}{2}(a + l) = \frac{n}{2}[2a + (n-1)d]}$

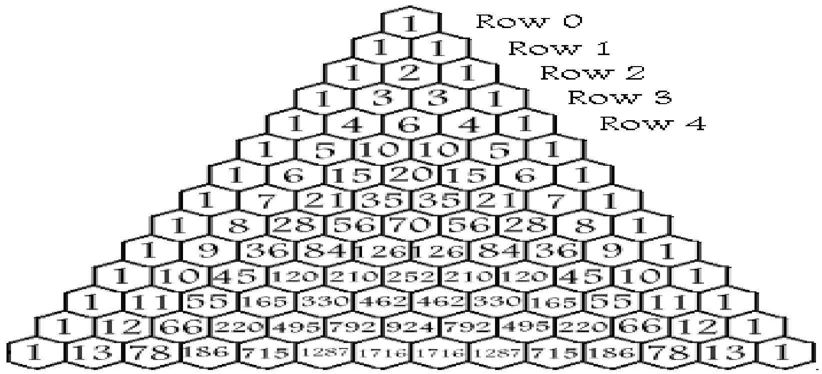
7.04 SUM OF G.P SERIES

The sum of the finite G.P is $\boldsymbol{S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n - 1)}{r-1}}$

N.B: A.P and G.P questions in K.C.S.E are mostly given as statements from which the candidate is required to identify the series, form the necessary equations after locating the correct terms, sums or both.

8.00 BINOMIAL EXPANSION

8.01 PASCAL'S TRIANGLE



Locate the coefficients from Pascal's triangle in your four figure tables.

8.02 COEFFICIENTS OF TERMS IN A BINOMIAL EXPANSION

Using the calculator, the coefficients of $(x + y)^n$ are $n_{C_0},$, $n_{C_1},$ $n_{C_2},$, $n_{C_3},$ $n_{C_4},$ up to the number of terms needed. Or use the Pascal's triangle for the coefficients.

8.03 COMPUTATION USING BINOMIAL EXPANSION

To expand $(x + y)^n$: where x if the first term and y the second term of the expression, proceed as follows:

Write down each coefficient up to the number of terms needed, each coefficient should have each binomial term individually in brackets. Sum these coefficients with the binomials. Write the brackets with the first binomial with decreasing exponents from n up to the terms needed. At the same time, the brackets with the second binomial are written with increasing exponents from zero up to the terms needed in the expression.

$(x + y)^n = n_{C_0}\, x^n + \ n_{C_1}\,(x)^{n-1}(y)^1 + n_{C_2}\,(x)^{n-2}(y)^2 + n_{C_3}\,(x)^{n-3}(y)^3 + n_{C_4}\,(x)^{n-4}(y)^4 + $ -

Example: Expand $\left(1 - \frac{1}{2}x\right)^8$ up to the term in x^3

$\left(1 - \frac{1}{2}x\right)^8 = \left(1 + -\frac{1}{2}x\right)^8$, coefficients are 1, 8, 28 and 56.

$1(1)^8\left(-\frac{1}{2}x\right)^0 + 8(1)^7\left(-\frac{1}{2}x\right)^1 + 28(1)^6\left(-\frac{1}{2}x\right)^2 + 56(1)^5\left(-\frac{1}{2}x\right)^3$
$= 1 - 4x + 7x^2 - 7x^3$

8.04 EVALUATION OF NUMERICAL CASES USING BINOMIAL EXPANSION.

To solve numerical problems of the form $(m.xy)^n$ rewrite the number in brackets suitably into a sum e.g $(m + .xy)$, then use binomial expansion to evaluate.

In the example above, the expression can be used to evaluate $(0.995)^8$.

$0.995 = 1 - \frac{1}{2}x$, $x = 0.001 \rightarrow (0.995)^8 = 1 - 4(0.001) + 7(0.001)^2 - 7(0.001)^3 = 0.9607$

9.00 FORMULAE AND VARIATION

9.01 DIRECT VARIATION

The ratio of variables is a constant, $\frac{y}{x}$ = k or $y = kx$, where k is a constant. $\frac{y_1}{x_1} = \frac{y_2}{x_2}$, if the equation and constant are not required in the question.

9.02 INVERSE VARIATION:

The product of variables is a constant, $yx = k$ *or* y = $\frac{k}{x}$ where k is a constant.

$x_1y_1 = x_2y_2$, if the equation and constant are not required.

9.03 JOINT VARIATION

Joint variation could involve direct variation, inverse variation or both. Quantities becomes products, quotients or both.

- A quantity P varies directly as square root R and inversely as the square of Q, is the same as: A quantity P is directly proportional to the square root of R and inversely proportional to the square of Q. This gives $\frac{PQ^2}{\sqrt{R}}$ = k. OR P = $\frac{k\sqrt{R}}{Q^2}$
- A quantity P varies directly as Q and as the square root of R implies $\frac{P}{Q\sqrt{R}}$ = k. OR P = $kQ\sqrt{R}$
- A quantity P varies inversely as Q and inversely as the square root of R means $PQ\sqrt{R}$ = k. OR P = $\frac{k}{Q\sqrt{R}}$
- NOTE: If one or the two quantities change by given percentages, effect the changes and get the new expression.

  Percentage effect on the old expression = $\left(\frac{new\ expression}{old\ expression} - 1\right)$ x 100.

9.04 PARTIAL VARIATION:

Quantities are added which could involve direct variation or inverse variation or both.

- A quantity P is partly constant and partly varies as the square of Q → $P = k_1 + k_2Q^2$
- A quantity P varies partly as Q and partly as the square root of Q → $P = k_1Q + k_2\sqrt{Q}$

9.05 FORMULAE:

To change the subject of a formula:

- Clear roots, fractions, and brackets (If there are any)
- Put the term(s) containing the new subject on one side of the equation and everything else on the other side.
- Simplify any terms, if you can, and factor out the new subject if necessary.
- Reduce the term containing the new subject to a single letter by either, dividing, multiplying or taking a root.
- Write the equation or formula with the subject on the left hand side.
  Note: Only a few steps in this strategy could be necessary to making a give letter the subject of a formula.

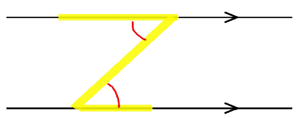
10.00 GEOMETRY

10.10 ANGLES AND PLANE FIGURES

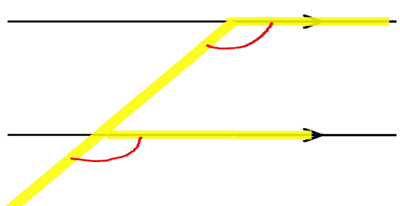
When you look at a figure, you should not assume segments or angles are congruent based on appearance. *Do not assume anything in Geometry is congruent – unless they are marked. This is also true for parallel & perpendicular lines.*

10.11 ANGLES ON A TRANSVERSAL

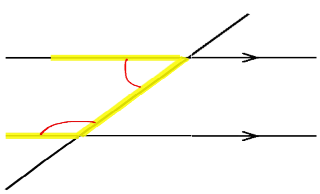
a) Alternate angles are equal. They are on alternate sides of the transversal; they can be recognised by the letter Z.



b) Corresponding Angles are equal. The angles are in the corresponding place out of the four pairs of angles, they can be recognised by the letter F.



c) Supplementary Angles add up to 180°. Supplementary angles can be recognised by the letter C.



Vertically Opposite Angles is equal.

10.12 ANGLE PROPERTIES OF POLYGONS

a) TRIANGLES

Interior angles of a triangle sum to 180°.

Angles on a line sum to 180°.

Base angles of an isosceles triangle are equal.

Isosceles triangles have two sides equal and two angles equal.

Equilateral triangles have three sides equal and three angles equal.

The acute angles of a right angled triangle are complementary.

The measure of an exterior angle of a triangle is equal to the sum of the measures of its remote interior angles.

b) REGULAR POLYGONS

All side lengths congruent or equal

All angles congruent

Exterior angle $e = \frac{360}{n}$

c) ALL POLYGONS

The sum of the interior angles of a polygon is given by the formula: $180(n-2) or\ 90(2n-4)$

Sum of exterior angles of a polygon is 360°,

$Sum\ of\ the\ exterior\ and\ interior\ angles$= 180 (angles on a straight line).

Pentagon (5 sides), Hexagon(6 sides), Heptagon or Septagon (7 sides), Octagon(8 sides),Nonagon or Novagon (9 sides),Decagon(10 sides).

10.13 ANGLES AT A POINT

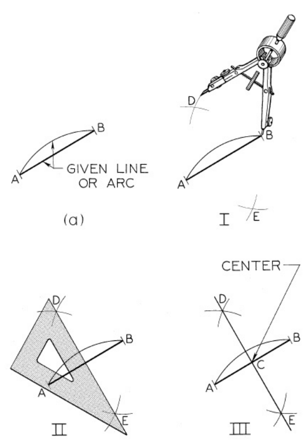
Angles at a point sum to 360°

10.20 GEOMETRIC CONSTRUCTIONS OF ANGLES AND TRIANGLES

CONSTRUCTION OF LINE AND ANGLES USING RULER AND COMPASSES ONLY

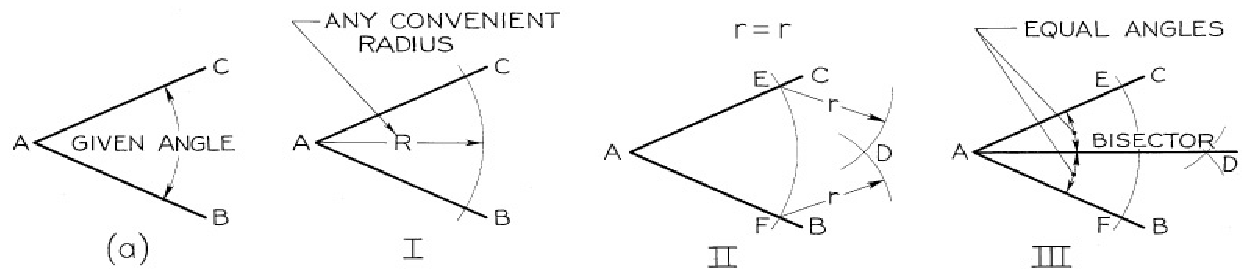
10.21 BISECTING LINE AB

Given the line A B, with points A & B as centers and any radius greater than ½ of AB, draw arcs to *intersect on both sides of the line AB*. The bisector passes through these points of intersection.



10.22 BISECTNG AN ANGLE

With point of intersection between the lines as the center and any convenient radius , draw an arc to intersect the two lines at any two points say A and B. With **A** and **B** as centers and any radius greater than ½ the radius of original arcs, draw two arcs to intersect at a point between the lines. The bisector passes through this point and the original point of intersection of the lines.



10.23. AN ANGLE OF 45°:

Bisect an angle of 90° that results from constructing a perpendicular bisector to a line.

10.24 AN ANGLE OF 60°:

Draw a line and pick two points on it. The distance between these two points will be your radius for the compass. Using this same radius; make two arcs from these points to intersect on one side of the line. Join up to form your angle. An angle of 30° is constructed by bisecting the angle of 60°.

10.25 CONSTRUCTION OF ANGLES OF MULTIPLES OF $\left(7\frac{1}{2}\right)^0$

Angles that are multiplies of $\left(7\frac{1}{2}\right)^0$ are constructed by bisecting angle of 30° or 45° and summing up to the angle required.

CONSTRUCTION OF PERPENDICULAR AND PARALLEL LINES USING RULER AND COMPASSES OR SET SQUARE ONLY

10.26 PERPENDICULAR TO A LINE THROUGH A GIVEN POINT ON THE LINE:

Place the pair of compasses at the point on the line; draw two arcs to cut the line on opposite sides of the point. Adjust the pair of compasses and using the two points where the arcs meet the line as centers draw two arcs that meet on one side of the line. Draw the straight line from this point to the original point on the line.

10.27 PERPENDICULAR FROM A POINT TO A LINE:

From the point, using a suitable radius draw an arc that cuts the line at two points. Adjust the pair of compasses and use the arcs that cut the line as centers. Using the same radius; make two arcs from these points to meet on one side of the line. Join this point to the original point.

10.28 PARALLEL LINE THROUGH A GIVEN POINT

Pick any two points on the line; the distance between them will be the radius for your compass. Using this radius make an arc from one of the points on the line. Using the same radius make an arc from the point through which the line is to pass. The arc made should meet the arc originally made. Draw your parallel line.

10.29 CONSTRUCTIONOF REGULAR AND IRREGULAR POLYGONS UPTO A HEXAGON

For constructions of triangles, sketch the triangle and then choose a suitable scale. Then follow the dimensions given for the construction.

Once the length and interior angle are known, construct one section of the polygon at a time until the polygon is complete.

10.30 LOCI

Locus is the path, area, or volume traced out by a point, line, or region as it moves obeying some given rule or rules.

10.31 THE PERPENDICULAR BISECTOR LOCUS:

Locus of points that is equidistant from two fixed points. This is the bisector of the line joining the fixed points.

10.32 PARALLEL LINES LOCUS:

The locus of points that is equidistant from a given straight line. This will be a set of parallel lines on opposite sides of the line.

10.33 CIRCLE LOCUS:

The locus of points that is equidistant from a fixed point is a circle, whose centre is the point, and the radius is the distance.

10.34 ANGLE BISECTOR LOCUS:

The locus of a point that moves as to be equidistant from two given intersecting lines will be the bisectors of the angles that the lines make.

10.35 CONSTANT ANGLE LOCUS:

This is the locus of points that move such that a line segment subtends a constant angle ϴ. Draw the perpendicular bisector of the chord. From any end of the chord construct the angle (90 - ϴ), this helps you locate the centre of the circle segments that lies on the perpendicular bisector of the chord. The circle segments end at the ends of the chord, but on opposite side of it.

10.36 INTERSECTING LOCUS:

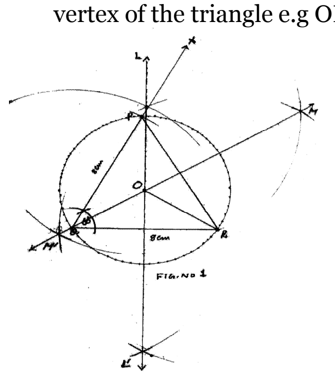
This could involve two or several loci that end up describing a region.

10.37 INEQUALITIES LOCI:

This could involve several of loci stated above or actual inequalities. A region is normally located in most cases.

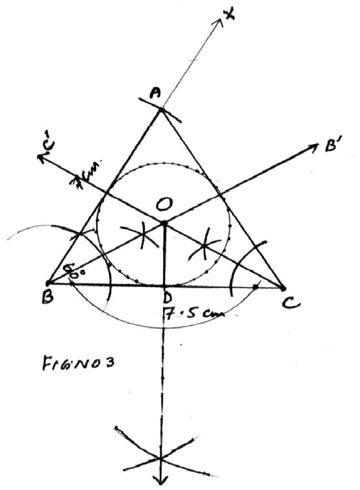
10.38 CONSTRUCTION OF CIRCUMSCRIBED OR CIRCUMCIRCLE (CIRCLE ROUND A TRIANGLE)

- Draw the perpendicular bisectors of at least two sides to locate the centre of the circle. The centre of the circle is at the intersection of the perpendicular bisectors. The radius of the circle is from the centre to any vertex of the triangle e.g OB.



10.39 CONSTRUCTION OF THE INSCRIBED CIRCLE (CIRCLE INSIDE THE TRIANGLE)

- Draw angle bisectors of at least two interior angles of the triangle.
- The point of intersection will be the centre of the circle.
- Construct a perpendicular line from the centre to any side of the triangle.
- The radius of the circle is from the centre to where the perpendicular line from the centre touches any side of the triangle e.g OD.



10.40 BEARINGS AND SCALE DRAWING

10.41 BEARINGS

Bearings are:

Measured from **North**, In a **clockwise** direction and Written as **3 figures.**

Use your protractor to measure the bearing of each point from the centre of the circle.

To find the bearing of B from A:

1. Draw a straight line between both points. 2. Draw a North line at A. 3. Measure the angle between them.

If the bearing of B from A is known as θ, then the bearing of A from B is given by

$180 + \theta$ for $\theta<180$ and $\theta - 180$ if $\theta > 180$.

10.42 Scale drawing:

First sketch what is to be drawn, then start scale drawing by following instructions with a suitable scale. A **scale** is the ratio between two sets of measurements. Scales can use the same units or different units.

A **scale drawing** is a proportional drawing of an object. A suitable or stated scale must be used in drawing.

All scale drawings **must** have a scale written on them. Scales are usually expressed as ratios. Normally for maps and buildings the ratio: **Drawing length: Actual length**

10.43 ANGLES OF ELEVATION AND DEPRESSION

If an observer sights an object above, the angle between the horizontal line and his or her line of sight is called an angle of elevation. If the observer sights an object below, the angle of the horizontal and his or her line of sight is called the angle of depression.

10.44 SURVEYING TECHNIQUES

The irregular shaped land is divided into small regular shaped figures mostly trapezoids and triangles by drawing offsets whose lengths are given in the field book.

$$\left\langle \begin{matrix} offsets \\ to\ the\ left \\ and\ their \\ perpendicar \\ distances \end{matrix} \middle| \begin{matrix} refence\ line \\ from\ which \\ off\ sets\ are \\ drawn. \\ Distances\ between \\ offsets\ is\ given \end{matrix} \middle| \begin{matrix} offsets \\ to\ the\ right \\ and\ their \\ perpendicular \\ distances \end{matrix} \right\rangle$$. From these figures the sketch of the plot can be done.

Distances between offsets, is the difference between consecutive values given on the reference line. From these details, the area of each trapezium or triangle can be found. The total area will be the sum of the areas.

10.50 COMMON SOLIDS

10.51 NETS OF SOLIDS

A ***net*** is a two-dimensional figure that, when folded, forms a three-dimensional figure. The net is the arrangement of the surfaces correspondingly. The net of a solid is drawn by tracing the faces of the solid as the solid is rolled forward or backwards and sideways.

DISTANCE BETWEEN TWO POINTS ON THE SURFACE OF A SOLID

Open up the solid into its net. The length of the straight line joining the two points gives the distance between the two points unless stated otherwise in the question. It is the shortest distance.

10.60 THREE DIMENSIONAL GEOMETRY

Remember: Pythagoras theorem, trigonometric ratios and sine and cosine rules.

10.61 ANGLE BETWEEN A LINE AND A LINE

The angle between two lines can be found by drawing out the suitable triangle concern. Solve the triangle for the angle.

10.62 ANGLE BETWEEN A LINE AND A PLANE

To find the angle between a line and a plane:

- Identify a suitable normal to the plane, one that touches the line.
- Identify and determine the orthogonal projection of the line onto the plane.
- Draw out the right angled triangle, name the angle and find its size.

10.63 ANGLE BETWEEN A PLANE AND A PLANE

To find the angle between two planes:

- Identify or construct two lines that meet on the line of intersection of the two planes, one on each plane. These two lines must be perpendicular to the line of intersection of the two planes.

- Identify and draw out a suitable triangle. Name the angle and find its size.

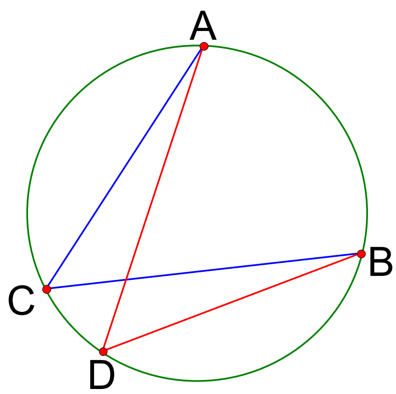
10.64 ANGLE BETWEEN SKEWLINES

Translate one of the lines to the plane containing the other line. Then find the angle between the two lines normally.

11.00 ANGLE PROPERTIES OF A CIRCLE

11.01 ANGLES SUBTENDED BY THE SAME ARC AT THE CIRCUMFERENCE

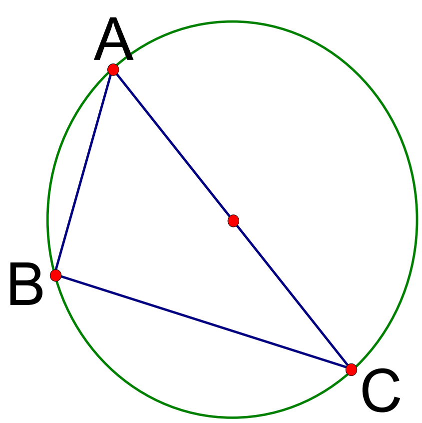
“Angles subtended by an arc in the same segment are equal”



Also angles subtended by equal chords are equal.

11.02 ANGLES IN A SEMI-CIRCLE

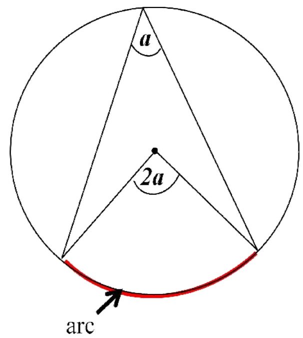
Angles formed by drawing lines from the ends of the diameter of a circle to its circumference form a right angle. We say *“The angle in a semi-circle is a Right Angle”*



11.03 RELATIONSHIP BETWEEN ANGLE SUBTENDED AT THE CENTRE OF A CIRCLE AND ANGLE SUBTENEDED ON THE CIRCUMFERENCE

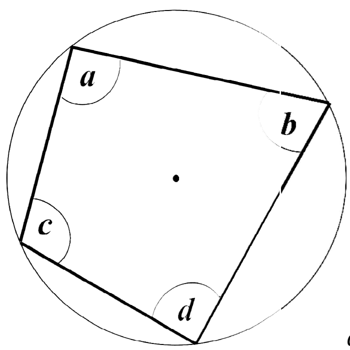
The angle formed at the centre of the circle by lines originating from two points on the circle's circumference is double the angle formed on the circumference of the circle by lines originating from the same points. We say “*If two angles stand on the same chord, then the angle at the centre is twice the angle at the circumference*”

The angle subtended by an arc at the centre of a circle is twice that at the circumference



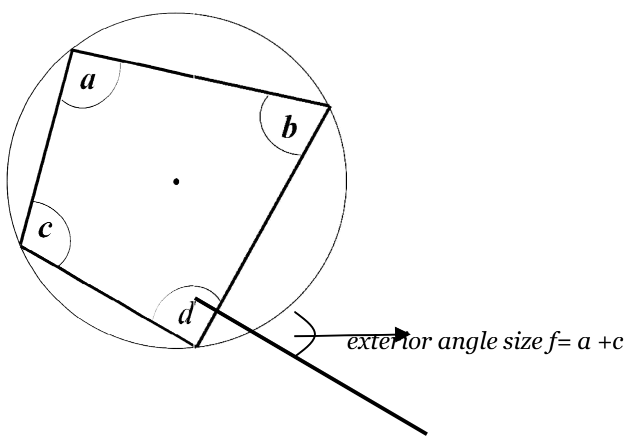
11.04 ANGLE PROPETIES OF A CYCLIC QUADRILATERAL

1. Opposite angles of a cyclic quadrilateral are supplementary



$a + d = 180^0$; $c + b = 180^0$

2. The exterior angle of a cyclic quadrilateral is equal to the sum of the interior opposite angles.



12.00 CIRCLES, CHORDS AND TANGENTS

12.01 LENGTHS OF TANGENTS AND CHORDS

Tangents: A tangent to a circle is a straight line which touches the circle at only one point.

A tangent to a circle forms a right angle with the circle's radius, at the point of contact of the tangent and the circle.

If two tangents are drawn on a circle and they cross, the lengths of the two tangents (from the point where they touch the circle to the point where they cross) will be the same.

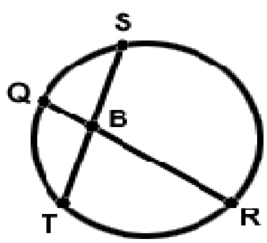
12.02 PROPERTIES OF CHORDS AND TANGENTS

1. A line joining the centre of the circle to the mid-point of a chord is perpendicular to the chord and bisects it. Any perpendicular bisector of chords will always pass through the centre of the circle.

2. If chords are equidistant from the centre of the circle, the chords are equal in lengths. These equal chords will subtend equal angles at the centre of the circle.

12.03 INTERSECTING CHORDS:

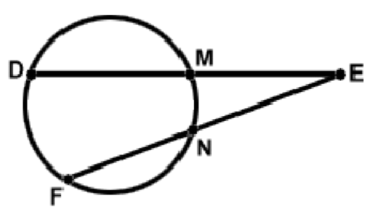
a) If two chords (or secants) intersect inside a circle, then the products of the lengths of the chord segments are equal. I f chords QR and ST intersect internally at B then $QB \cdot BR = SB \cdot BT$



An <u>external secant segment</u> is a secant segment that lies in the exterior of a circle.

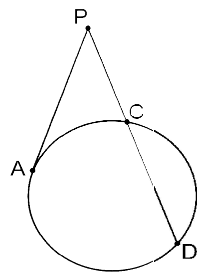
b) If two secants intersect outside of a circle, then the product of one segment length and its external secant segment length is equal to the product of the other secant length and its external secant segment length.

- $DE \times EM = FE \times EN$



c) If a secant and a tangent intersect outside the circle, it is very similar to two secants. The tangent is both the 'exterior segment' and the 'whole segment'. Hence the product of one segment length and its external secant segment length is equal to the square of the tangent length.

- $(PA)^2$ = DP×PC



12.04 CONTRUCTION OF AN ESCRIBED CIRCLE

To ascribe a circle to a given side of a triangle e.g AC, bisect the two external angles of the triangle to locate the centre of the circle, O. Drop a perpendicular from this point to the side of the triangle or the extended side. This gives you the radius of the circle, OD.

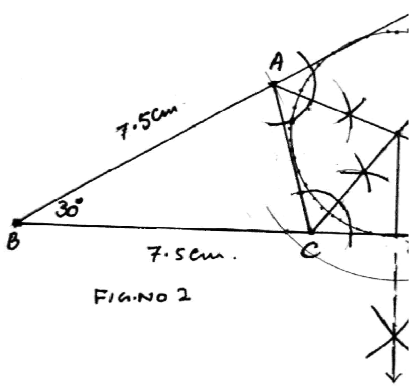
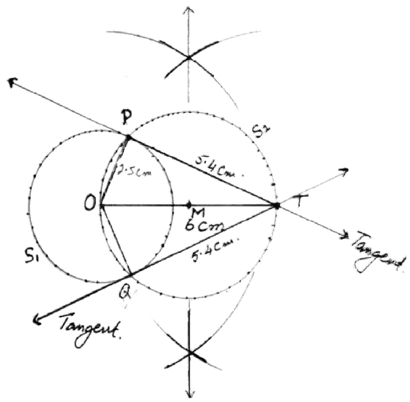


FIG.NO 2

N.B: The side bisectors of a triangle joined to the opposite vertexes (lines called medians) meet at the centroid of the triangle. Dropping perpendiculars from the vertexes of a triangle to the opposite sides locates the orthocenter of the triangle.

12.05 CONSTRUCTION OF TANGENTS TO A CIRCLE

Let the circle have centre O and the external point be T. Join T to O. Bisect TO to locate its mid-point M. With M the centre and radius MT or MO, draw a circle to intersect the original circle at P and Q. PT and QT will be the tangents.



12.06 DIRECT COMMON TANGENTS TO TWO CIRCLES

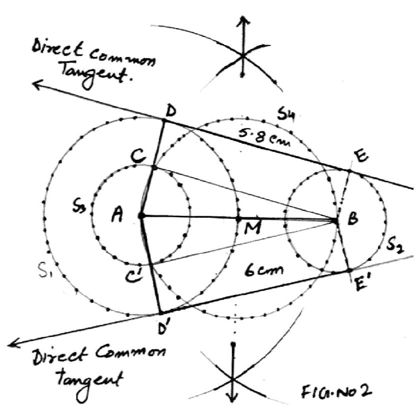
Let the two circles have the centers A for the larger circle and B for the smaller circle.

Join A to B. Draw a concentric circle with centre A and radius = difference of radii.

Bisect the line AB to locate midpoint of AB as centre M of another circle. With M as centre and radius MA or MB draw circle to cut smaller concentric circle at points C and C' .

Join A to C and A to C' both produced to meet large circle at D and D' respectively.

Join B to C and B to C'. With centre D and radius CB make an arc to cut the original small circle at E. With centre D' and radius CB make an arc to cut the original smaller circle at E'. DE and D'E' are the tangents.



12.07 INVERSE COMMON TANGENTS TO TWO CIRCLES

Let the two circle have centre A for the larger circle and B for the smaller circle. Join A to B. With A as the centre draw a circle with radius = sum of radii.

Bisect AB to locate mid-point M on it. With M as the centre and radius MB draw a circle which intersects the circle whose radius = sum of radii at points C and C'.

Then Join A to C and A to C' to intersect the large circle at points D and D'.

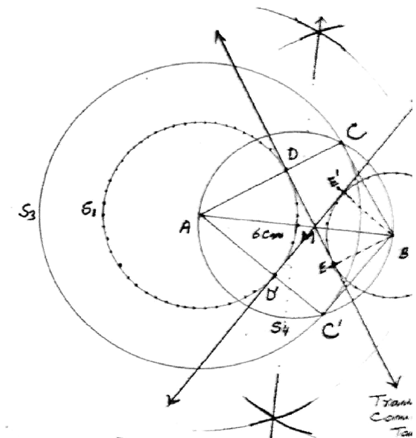
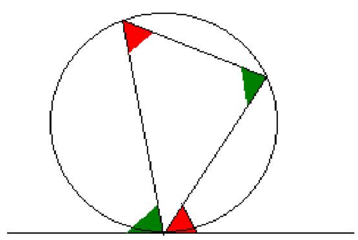


FIG NO 3

Join B to C and B to C'. With center D and radius BC draw an arc to intersect the smaller circle oppositely at E. With centre D' and radius BC draw an arc to intersect the small circle oppositely at E'. DE and D'E' are your tangents.

12.08 ANGLES IN THE ALTERNATE SEGMENT

The angle between a tangent to a circle and a chord drawn through the point of contact is equal to any angle subtended by the chord at the circumference in the alternate segment.



13.00 GRAPHS

13.10 COORDINATES AND GRAPHS

13.11 CHOICE OF SCALES

If the scale has not been given, choose suitable scale(s). Use the range of values of x and y and the space provided to choose a scale. The scale(s) must be uniform and simple. Use scale(s) that make plotting decimals easy.

13.12 TABULATION OF VALUES FOR A LINEAR RELATION

Find the (x, y) values for three points only. Two of these points give you the line. The third acts as a check.

13.13 GRAPHICAL SOLUTION OF LINEAR SIMULTANEOUS EQUATIONS

The two linear relations are drawn on the same axes. The coordinates of the point of intersection of the two lines satisfy both equations simultaneously. The x and y values will be the solutions for the set of simultaneous equations.

13.20 GRAPHICAL METHODS

13.21 TABLES OF VALUES FOR GRAPHS OF A GIVEN RELATION

A table of values to be completed is normally given. Use your calculator carefully, especially when dealing with negative values, decimals and fractions to complete the table. Make proper use of brackets in your calculator.

13.22 GRAPHS OF QUADRATIC AND CBIC RELATIONS

Write down the (x, y) coordinates of the points from your table of values. Choose a suitable scale or use the given scale to plot the points with the coordinates you have listed. Mark the points with a cross or circled dot. Use your free hand to draw the curve through the points. Label the curve with its equation.

13.23 SOLUTIONS OF QUADRATIC AND CUBIC EQUATIONS FROM GRAPHS

The equation of a straight line to be drawn must be determined. This is done by making the coefficients of x^3, x^2 or both in the equations equal by multiplying all terms by a constant in either the graph equation or the equation to be solved or both.

Then the difference between the adjusted graph equation and the equation to be solved gives the equation of the straight line to be drawn. The solutions of the given equation are the values of x at the point(s) where the line meets the curve

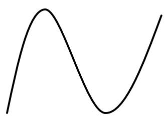
Graphs meet at points on the Cartesian plane where they have simultaneous solutions.

13.24 QUADRATIC CURVES

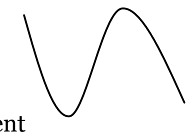
- If the coefficient of x^2 is positive the graph is a cup (has a minimum). If the coefficient is negative the graph will be a cap (has a maximum).
- If a< 0, b^2 > 4ac, the cap crosses the x – axis.
- If a< 0, b^2 = 4ac, the cap touches the x – axis and is below the axis.
- If a< 0, b^2 < 4ac, the cap below the x- axis
- If a> 0, b^2 > 4ac, the cup crosses the x – axis.
- If a> 0, b^2 = 4ac, the cup touches the x – axis and is above the axis.
- If a> 0, b^2 < 4ac, the cup is above the x- axis

13.25 CUBIC GRAPHS

Cubic curves have the features shown below, depending on the sign of the coefficient of x^3.



Positive coefficient



negative coefficient

13.26 AVEARAGE AND INSTANTANEOUS RATE OF CHANGE

Average rate of change is the gradient of a line joining two points on a curve.

The instantaneous rate change is the gradient of a tangent to the curve at a point.

13.27 CHANGING OF NON-LINEAR LAWS TO LINEAR LAWS

Directly or adjust the equation to compare with Y = mX + c. For example:

1. $y = ax^2 + b$, compares with Y = mX + c, plot y against x^2. m = a, c = b.

2. $y = a\sqrt{x} + b$, compares with Y = mX + c, plot y against $\sqrt{x}$. m = a, c = b.

3. $y = \frac{a}{x} + b$, compares with Y = mX + c, plot y against $\frac{1}{x}$, m = a, c = b.

4. $y = \frac{a}{x^2} + b$, compares with Y = mX + c, plot y against, $\frac{1}{x^2}$, m = a, c = b.

5. $y = ax^2 + bx$, adjust to $\frac{y}{x} = (ax) + b$, plot $\frac{y}{x}$ against x, m = a, c = b.

6. $y = \frac{a}{x} + bx$, adjust to $xy = a + bx^2$, plot xy against x^2, m = b, c = a.

7. $y = ba^x$, adjust to logy = logb + xloga. Plot logy against x. m = loga, c = logb.

8. $y = ax^n$, adjust to logy = loga + nlogx. Then Plot logy against logx. m = n, c = loga.

9. $y = \frac{1}{ax^2+b}$, adjust to $\frac{1}{y} = ax^2 + b$, plot $\frac{1}{y}$ against x^2. m = a, c = b.

10. y = ax + b/x, adjust to $xy = ax^2 + b$, plot xy against x^2. m = a, c = b.

11. $y = \frac{a}{x} + (b\sqrt{x})$, adjust to $xy = a + bx\sqrt{x}$, plot xy against $x\sqrt{x}$. m = b, c = a.

12. $\frac{a}{y} = \frac{b}{x} + 1$, adjust to $\frac{1}{y} = \frac{b}{a}\left(\frac{1}{x}\right) + \frac{1}{a}$, plot $\frac{1}{y}$ against $\frac{1}{x}$, m = $\frac{b}{a}$, c = $\frac{1}{a}$.

13. $y = ax^2 + x + b$, adjust to $y - x = ax^2 + b$, plot $y - x$ against x^2. m = a, c = b.

14. $y = x^2 - ax + b$, adjust to $y - x^2 = -ax + b$, plot $y - x^2$ against x. m = -a, c = b.

13.28 BEST LINE OF FIT, EMPIRICAL DATA AND THEIR GRAPHS

N.B: In graphical methods you could be required to draw a graph directly from a table. The Best line of fit is always required in a question to approximate to linear laws. As many points as possible should be on the line, and the numbers of points on either side of the line should approximately be the same.

13.29 EQUATIONS OF A CIRCLE, THE CENTRE AND RADIUS OF A CIRCLE

1. $x^2+y^2 = a^2$ is the equation of a circle centre (0,0) and radius a.

2. The coefficients of x^2 and y^2 must be the same for the equation of the circle.

3. To locate the centre of the circle and determine its radius, ensure that the coefficients of x^2 and y^2 are each equal to one and then proceed as follows:

- $x^2 + y^2 + ax + by + c = 0$, Centre of the circle C $\left(-\frac{a}{2}, -\frac{b}{2}\right)$,
  r = $\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2 - c}$
- A line that bisects a chord at right angles passes through the centre of the circle.
- The centre of the circle is the mid-point of any diameter.

14.00 TRIGONOMETRY

14.10 TRIGONOMETRY I

14.11 TANGENT, SINE AND COSINE OF ANGLES

SOHCAHTOA, OSHACHOTA can be used to remember the trigonometric ratios. **S**ome **O**ld **H**orses **C**an **A**lways **H**ear **T**heir **O**wn **A**pproach. **S**ome **O**f **H**er **C**hildren **A**re **H**aving **T**rouble **O**ver **A**lgebra

Always draw out the right angled triangle to be solved and apply the above statement. Angle of elevation is located horizontal up to the line. The angle of depression is located horizontal down to the line.

14.12 SINE AND COSINE OF COMPLEMENTARY ANGLES

$\sin\theta = \cos(90 - \theta)$ where θ is acute.

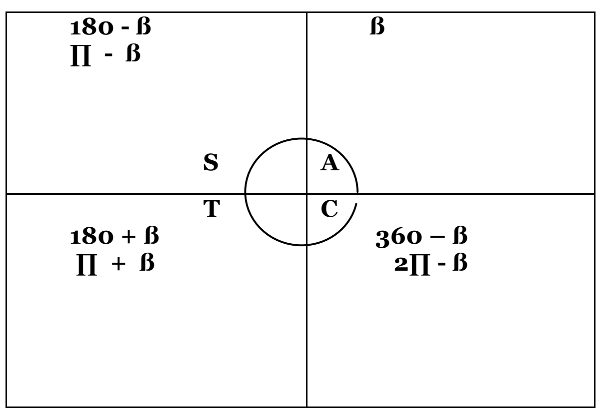
Given one trigonometric ratio of a triangle, the other trigonometric ratios can be found by sketching the right angled triangle involved, find the other side using Pythagoras theorem. With the sides known, the other trigonometric ratios can be determined.

14.13 TRIGONOMETRIC RATIOS OF SPECIAL ANGLES

Exact trigonometric ratios of common angles of 30° and 60° can be found using right angled triangles of angles 30°, 60° and sides $\sqrt{3}$, 2 and 1 in lengths. The trigonometric ratios for 45° use a right angled triangle of sides 1, 1, and$\sqrt{2}$. (Longest side will always be the hypotenuse.)

14.20 TRIGONOMETRY II

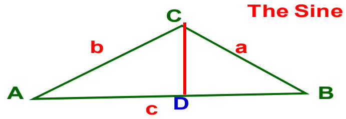
14.21 TRIGONOMETRIC RATIOS OF ANGLES GREATER THAN 90° AND NEGATIVE ANGLES (INCLUDING THE RADIAN MEASURE)



Identify the quadrant in which the angle is. Equate the angle to the suitable expression in that quadrant so as to find the acute angle ß. Use this acute angle to find the value of the trigonometric ratio. Do not forget to insert the relevant sign on the value, in relation to the quadrant. The calculator can be used directly to fine the trigonometric ratios.

14.22 SINE RULE: for any triangle, the ratio of a side to the sine of its opposite side is the same for all three sides.

The Sine Rule



CD = b sin A
CD = a sin B

Equating the two values of CD we have:

a sin B = b sin A   Divide both sides by sin A sin B

$\frac{a}{\sin A} = \frac{b}{\sin B}$ Similarly we can show that this ratio is also equal to $\frac{c}{\sin C}$

This gives the sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

For any triangle, the ratio of a side to the sine of its opposite side is the same for all three sides.

$\frac{a}{sin\ A} = \frac{b}{Sin\ B} = \frac{c}{sin\ C} = 2R$ where R is the radius of the circumscribed circle round the triangle ABC.

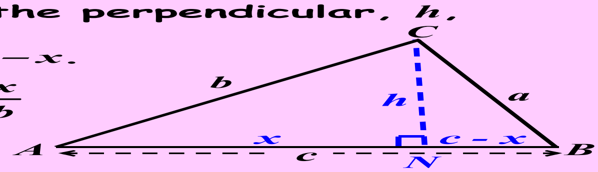
We use sine rule when we have two angles and a side (AAS) or We have two sides and the non-included angle (this is the ambiguous case)

14.23 **COSINE RULE FOR TRIANGLE ABC**: $a^2 = b^2 + c^2 - 2(b)(c)(cosA)$

The Cosine Rule

Proof of the Cosine Rule

In the triangle ABC, draw the perpendicular, h, from C to AB.
Let $AN = x$.   Then, $NB = c - x$.



From triangle ANC, $\cos A = \frac{x}{b}$

$\Rightarrow \quad x = b\cos A \quad - - - - - \ (1)$

Using Pythagoras' theorem:

In triangle ANC, $h^2 = b^2 - x^2$

In triangle BNC, $h^2 = a^2 - (c - x)^2$

So, $b^2 - x^2 = a^2 - (c - x)^2 \quad - - - - - \ (2)$

The Cosine Rule

Proof of the Cosine Rule

We have $\quad b^2 - x^2 = a^2 - (c - x)^2 \quad - - - - - \ (2)$

Simplifying: $\quad b^2 - x^2 = a^2 - (c^2 - 2cx + x^2)$

$\Rightarrow \quad b^2 - x^2 = a^2 - c^2 + 2cx - x^2$

$\Rightarrow \quad b^2 = a^2 - c^2 + 2cx$

Substituting for x from equation (1), ($x = b\cos A$)

$\Rightarrow \quad b^2 = a^2 - c^2 + 2cb\cos A$

Rearranging:

$b^2 + c^2 - 2bc\cos A = a^2$

$\Rightarrow \quad \boxed{a^2 = b^2 + c^2 - 2bc\cos A}$

We use cosine rule when we have two sides and the INCLUDED angle (SAS) or all three sides (SSS). The letters can be switched to find any side provided it is opposite the given angle. If we want to find an angle, we use the sine rule after we have used the cosine rule.

14.30 TRIGONOMETRY III

14.31 GRAPHS OF TRIGONOMETRIC FUNCTION OF THE FORM

$(\boldsymbol{a})\boldsymbol{f}(\boldsymbol{bx}+\boldsymbol{c})$ Where, f can be any of the three trigonometric functions.

Table of values are normally completed before the graphs are drawn. Make proper use of your calculator for the completion of the table of values. Sometimes students must choose suitable scales for the graph, such scales must be simple and uniform and within the given range.

If a trigonometric graph crosses the x – axis, then the solutions of the graph with y = 0 will be the x - coordinate of such intersections.

If two trigonometric graphs are plotted on the same axis, the values of x at the points of intersections will be the solutions to the equations that result from the two trigonometric functions being equated. Check the equations you are requested to solve after plotting, they will always arise from equating the plotted trigonometric functions.

Plotted trigonometric function could be related to a constant c using an inequality, draw or locate the line y = c. The values of x at the points of intersection of this line and the trigonometric curve, gives you the range(s) of the values of x.

Some inequality could involve one curve being over another. The values of x at the point of intersection between these curves will easily lead you to the solution of the given inequality.

14.32 SIMPLE TRIGONOMETRIC EQUATIONS

1. If $\sin\theta = a$, then $\theta = (-1)^n \sin^{-1} a + 180n$ OR $\theta = (-1)^n \sin^{-1} a + \pi n$

2. If $\cos\theta = a$, then $\theta = \pm \cos^{-1} a + 360n$ OR $\theta = \pm \cos^{-1} a + 2\pi n$

3. If $\tan\theta = a$, then $\theta = \tan^{-1} a + 180n$ OR $\theta = \tan^{-1} a + \pi n$

Where $n = 0, 1, 2, 3, 4, etc\ \ and\ \ the\ principal\ angle\ can\ be\ positive\ or\ negative$

Some trigonometric equations must be reduced to quadratic form using the identity

$\boldsymbol{cos^2\theta + sin^2\theta = 1}$.

14.33 AMPLITITUDE, PERIOD, WAVELENGTH AND PHASE ANGLE OF TRIGONOMETREIC FUNCTIONS

If a trigonometric function has been stated as y = $af(bx+c)$. a is the amplitude, period T = $\frac{360}{b} = \frac{2\pi}{b}$. The **period** is the distance (or time) that it takes for the sine or cosine curve to begin repeating again.

The **phase shift** is the amount that the curve is moved in a horizontal direction from its normal position, the displacement will be to the **left** if the phase shift is negative, and to the **right** if the phase shift is positive. Phase shift is found by equating $bx + c\ to\ 0$.

15.00 COMMERCIAL ARITHMETIC

15.10 COMMERCIAL ARITHMETIC I

15.11 PROFITS AND LOSS

A markup is an amount by which a wholesale cost is increased. A retail store buys items at *wholesale prices*. To cover expenses and make a profit, they sell items at higher *retail prices*. The extra amount is called the markup. Mark up is a percentage of the wholesale price/cost. Final price = markup + wholesale cost.

Percentage profit = $\frac{profit}{buying\ price}$ x 100. Percentage loss = 100 x $\frac{loss}{buying\ price}$.

Profit = selling price – buying price. Loss = Buying price – selling price.

15.12 EXCHANGE RATES

The bank buys and sells foreign currency at the prices indicated on the question paper. The selling price is higher than the buying price, because the bank has to make profit.

15.13 DISCOUNTS

A discount is an amount by which an original price is reduced. When an item is on *sale*, the store is selling the item for less, so this is called discount. Giving discounts does not equal to selling at a loss. The profit margin is actually reduced.

Discount is a percentage of the original price. Final price = original price – discount.

15.14 COMMISSIONS

Commission is a percentage of the amount earned as indicated in the question. Different amounts earned could attract different commissions. The Owner or Company receives the balance after paying commissions**.**

Total earnings = Salary + total commissions.

15.20 COMMERCIAL ARITHMETIC II

15.21 SIMPLE INTEREST AND CALCULATION

SIMPLE INTERST FORMULA : $(P \times R \times T)$=(I) = $I = (A - P)$ OR A = P + I .

A = amount accumulated, P = Principal.

A = P (1 +RT) where R is interest rate as a decimal, T is number of interest periods.

15.22 COMPOUNG INTEREST FORMULA AND CALCULATION

(A) = (P) + $\left(1 + \frac{r}{n}\right)^{nt}$, where A is Amount, P is Principal or Amount at start, r is the annual interest rate as a decimal, t is the time in years, and n is the number of times per year that interest is compounded or number of conversion periods or number of compounding periods per year.

15.23 COMPOUND INTEREST EQUATION FOR MULTIPLE INVESTMENTS IN EQUAL AMOUNTS

$(A) = (PMT) \times \frac{(1+r)^n - 1}{r}$ where PMT is the payments, r = interest rate as a decimal,

n = interest periods and A is the investment worth.

A step by step method can be used to get the Amount and interest.

15.24 APPRECIATION AND DEPRECIATION (USE OF COMPOUND INTEREST FORMULA)

APPRECIATION: Increase in value of any asset. A = P $(1 + r)^n$ where r = appreciation rate as a decimal and n = appreciation periods. Inflation is increase in value, i.e appreciation.

DEPRECIATION: Decrease in value of any asset. The above formula becomes:

A = P $(1 - r)^n$

15.25 HIRE PURCHASE

Hire purchase deposit = cash price x deposit percentage given.

Hire purchase price = cash price + total hire purchase interest.

Hire purchase price = Deposit + $(Monthly\ instalments)(No.of\ months)$

Hire purchase interest per installment = $\frac{Total\ hire\ purchase\ interest}{Number\ of\ total\ instalments}$

Equal installments = $\frac{Hire\ purchase\ price - Downpayment}{No.of\ instalments}$

$(n \times A) = (C - D)(1 + R)^n$ where n is the number of installments, A is the amount per installment, C is the cash value of the item, D is the deposit, and R is the rate as a decimal the item is compounded on.

15.26 INCOME TAX

Gross Income = salary + any other allowances and benefits due.

Taxable income = Gross Income – expenses incurred solely in the performance of one's duties.

If an employee is housed by the employer, freely or for nominal rent, then tax is levied on a salary equal to **115% basic salary less rent paid.**

After the payable tax is calculated, reliefs can be effected and other deductions.

16.00 STATISTICS AND PROPBABILITY

16.10 STATISTICS I

16.11 MEASURES OF UNGROUPED DATA

1. Mean for ungrouped data $\bar{x} = \frac{\sum x}{n} = \frac{sum\ of\ the\ values\ in\ the\ data}{total\ number\ of\ values}$

If a is assumed mean, then: $\overline{x} = \left(a + \frac{\sum t}{n}\right)$ where t = x − a.

2. Mode is the number that appears the most number of times, with the highest frequency.

3. The median is the middle number.

- To find the median, arrange the numbers in ascending order.
- Locate the middle number.
- If there are two middle numbers, the median is the mean of these two numbers

16.12 MEASURES OF GROUPED DATA

1. Mean for grouped data (locate the mid-point for respective classes say x) , $\overline{x} = \frac{\sum fx}{\sum f}$

2. Modal class is the class interval with the highest frequency.

REPRESENTATION OF DATA

16.13 PIE CHART

Pie charts are made of a circle divided into sectors made of angles gotten by $\frac{x}{total\ frequency}$ x 360.

16.14 HISTOGRAM

The range of possible values which can be put into each class is called the class interval. The class boundaries for each class are the smallest and largest values that an item in that class can have.

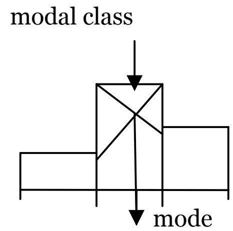
In a Histogram:

Area of bar is proportional to the frequency

Bars are joined together

Class boundaries mark the boundaries of the rectangular bars. Class intervals could differ. When class interval is doubled, the frequency is halved.

The modal class in a histogram is one with the highest frequency, and has the mode.



16.15 FREQUENCY POLYGONS

Constructed when the midpoints of the bar tops of the histogram are joined with a straight line. The frequency polygon can also be obtained by plotting frequency against the midpoints of the classes.

- Frequency density = $\frac{frequency}{class\ width}$, lower class boundaries on the x –axis against the frequency density make the histogram.

16.20 STATISTICS II

16.21 MEAN USING ASSUMED MEAN

1. When an assumed mean 'a' is used $\bar{x} = a + \frac{\sum t}{n}$ Or $\bar{x} = a + \frac{\sum ft}{\sum f}$ where t = x - a

2. For grouped data, go for the mid-point of every class say, x. where t = x – a. then

$\bar{x} = \left(a + \frac{\sum ft}{\sum f}\right)$

16.22 MEAN BY CODING

The grouped data can be tabulated as x, x – a, t = $\frac{x-a}{c}$, f and ft. Then $\sum f$ and $\sum ft$ are used to find the actual mean $\bar{x}$, as $\bar{x} = (ca) + \left(c\left(\frac{\sum ft}{\sum f}\right)\right)$

16.23 CUMULATIVE FREQUENCY TABLES AND OGIVE

For an Ogive or Cumulative frequency curve, plot cumulative frequency from the cumulative frequency table on the vertical axis against upper class limits in each case on the horizontal axis. This gives you an elongated S for a curve.

The Median and quartiles, and percentiles can be estimated from the ogive.

16.24 MEDIAN AND QUARTILES BY CALCULATION

The median class interval is the class interval containing the median. This must be at

$\frac{1}{2}\sum f$ of the cumulative frequency.

- Median = $L_m + \frac{\left(\frac{1}{2}(\sum f)\right) - (cf_a)}{f_m}$ x c_i ,where L_m is the lower class boundary of the median class, cf_a is the cumulative frequency above the median class, f_m is the frequency of the median class and c_i is the class interval of the median class.

16.25 RANGE, QUARTILES AND INTERQUARTILE RANGE

Lower quartile Q_1 is the 25% value of the distribution. The upper quartile Q_3 is the 75% value of the distribution. The inter-quartile range is the difference between Q_3 and Q_1. i.e $Q_3 - Q1$.

Inter-quartile range = upper quartile – lower quartile.

Semi interquartile range is half of the interquartile range.

Semi-interquartile range = $\frac{1}{2}(upper\ quartile - lower\ quartile)$

Deciles come about as a result of dividing the distribution into 10 equal parts. Percentiles are achieved when the distribution is divided into 100 equal parts. The values can easily be obtained from a cumulative frequency curve.

16.26 QUARTILE DEVIATIONS AND VARIANCE

Mean deviation is the mean of the deviations of the values from the mean. The deviations from the mean are taken as absolute values.

1. The Variance $б^2 = \frac{\sum(x-\bar{x})^2}{n}$ OR = $\frac{\sum f(x-\bar{x})^2}{\sum f} = \frac{\sum fx^2}{\sum f} - (\bar{x})^2$. The x, f, fx and (fx^2) are tabulated and $б^2$ can be found.

2. An assumed mean can be used to determine standard deviation. The grouped data can be tabulated as x, x – a, t = $\frac{x-a}{c}$, f and ft. Then $б^2 = c^2\left[\frac{\sum ft^2}{\sum f} - \left(\frac{\sum ft}{\sum f}\right)^2\right]$ = Variance.

16.27 STANDARD DEVIATION

The square root of the variance is equal to the standard deviation.

16.30 PROBABILITY

DEFINATION OF PROBABILITY

A probability is a measure of the likelihood that an event in the future will happen. It will always be a number between 0 and 1.

A value near zero means the event is not likely to happen. A value near one means the event is likely to happen.

An outcome is the particular result of an experiment.

An event is the collection of one or more possible outcomes of an experiment.

16.31 SAMPLE SPACE

The sample space of an experiment is the collection of all possible outcomes of an experiment. The set of all possible outcomes of an experiment or action is called the sample space. An outcome table is a useful way to show all the possible outcomes when two sample spaces are combined. Each entry in the table shows an outcome in the combined sample space.

Throwing a die twice or throwing two dice would result into the following outcome table:

	die 1 or first throw						
die 2 or throw 2		1	2	3	4	5	6
	1	**1,1**	**1,2**	**1,3**	**1,4**	**1,5**	**1,6**
	2	**2,1**	**2,2**	**2,3**	**2,4**	**2,5**	**2,6**
	3	**3,1**	**3,2**	**3,3**	**3,4**	**3,5**	**3,6**
	4	**4,1**	**4,2**	**4,3**	**4,4**	**4,5**	**4,6**
	5	**5,1**	**5,2**	**5,3**	**5,4**	**5,5**	**5,6**
	6	**6,1**	**6,2**	**6,3**	**6,4**	**6,5**	**6,6**

$\mathrm{P}(Event) = \frac{No.of\ desired\ outcomes}{Total\ no.of\ outcomes}$

COMBINED EVENTS

Two or more simple events can occur together (or one after the other) to give combined events.

16.32 INDEPENDENT EVENTS

- This is where one event has no influence on the outcome of another event. Two events are independent of each other if an occurrence in one event does not change the probability of an occurrence in the other.
- if events A and B are independent then P(A and B) = P(A) ×P(B)
- If P(A and B) = P(A) ×P(B), then events A & B are independent .
- Two or more events are said to be dependent if the occurrence or non-occurrence of one of the events affects the probabilities of occurrence of any of the others.
- The words replaced or with replacement will often give you a clue that independent events are taking place.

16.33 MUTUALLY EXCLUSIVE EVENTS:

Events are mutually exclusive if the occurrence of any one event means that none of the others can occur at the same time. The events cannot happen at the same time.

- if A and B are mutually exclusive events:

P(A or B) = P(A) + P(B)

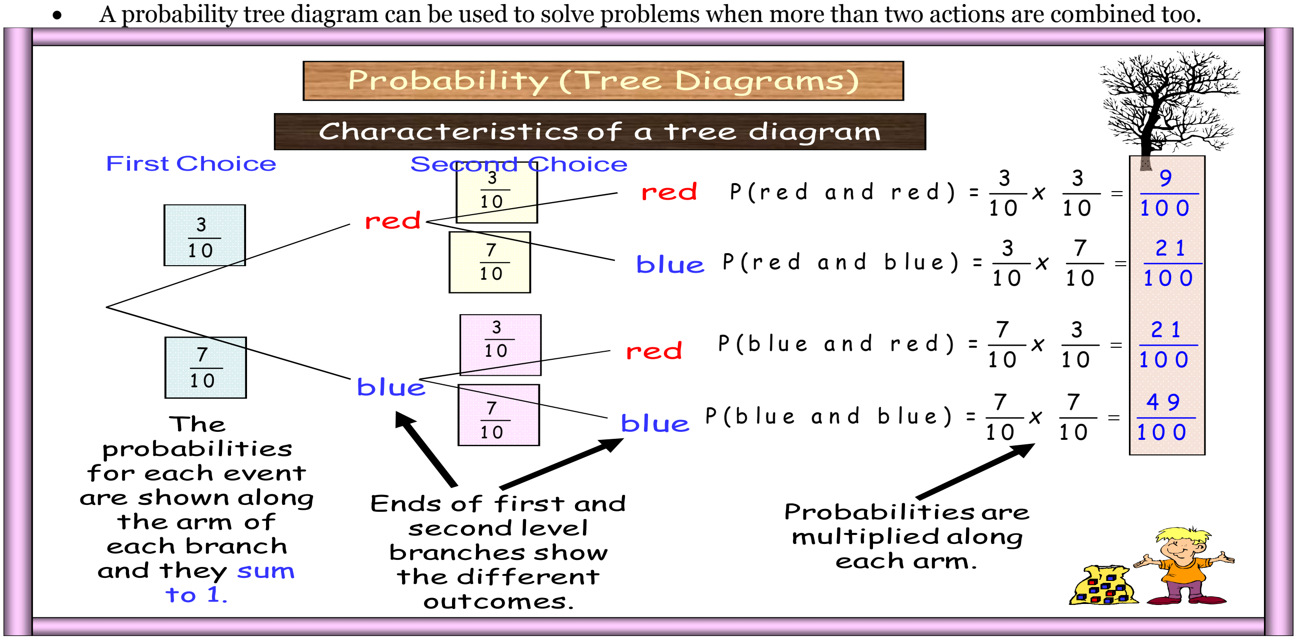
ex., die roll: P (1 or 6) = 1/6 + 1/6 = .33

16.34 PROBABILITY TREE DIAGRAMS

These can be used to solve questions involving combined events.

- Possible outcomes or events are shown at the end of a branch of a tree.
- Each action is shown by a ‘stage’ in the diagram. Two actions will mean two stages.

- Write the final outcome at the end of each route along the branch.
- Write each probability on each branch of the tree. Check whether the events are independent or not.
- Probabilities on adjacent branches must total to 1.
- The final outcome's probability is found by multiplying together the probabilities from any branch you go along.
- An event may involve more than one of these final outcomes. To find the probability that the event happens, add together the probabilities of these final mutually exclusive outcomes.
- A probability tree diagram can be used to solve problems when more than two actions are combined too.



N.B: P ($\bar{A}$) = P (Not A) = 1 – P (A)

17.00 VECTORS

17.10 VECTORS I

VECTOR NOTATION

Vectors are represented by directed lines.

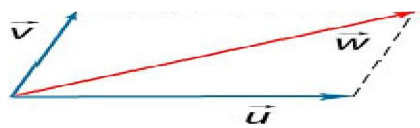
EQUIVALENT VECTORS

Vectors are equivalent or equal if they have equal magnitude or size and have the same direction.

17.11 ADDITION AND SUBTRACTION OF VECTORS

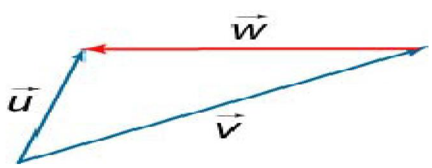
1. Vectors are about choosing alternative suitable routes as you move from a stated point to another. These routes must be made of known vectors or vectors that can easily be found.

2. If the end point of one vector is the starting point of another vector, the two vectors add up.



$\vec{u} + \vec{v} = \vec{w}$

If the starting points or the end points of the vectors coincide vectors subtract. Or moving against the direction of a vector is the same as subtracting that vector.



$\vec{u} - \vec{v} = \vec{w}$

17.12 COLUMN VECTORS

Vectors can be represented by column vectors like $\vec{a} = \binom{x}{y}$

17.13 MAGNITUDE OF A VECTOR

The magnitude or size of vector $\vec{a} = |\vec{a}| = \sqrt{x^2 + y^2}$

EQUIVALENT VECTORS

If column vectors are equal their corresponding elements are equal.

17.14 ADDITION AND SUBTRACTION OF COLUMN VECTORS

To add or subtract column vectors add or subtract corresponding elements in the column vectors.

$\vec{u} = \binom{a}{b}$ and $\vec{v} = \binom{c}{d}$ then $\vec{u} + \vec{v} = \binom{a}{b} + \binom{c}{d} = \binom{a+c}{b+d}$; $\vec{u} - \vec{v} = \binom{a}{b} - \binom{c}{d} = \binom{a-c}{b-d}$;

17.15 MULTIPLICATION OF A VECTOR BY A SCALAR

k $\binom{a}{b}$ = $\binom{ka}{kb}$ = $\vec{w}$, this means $\vec{w}$ is parallel to $\vec{u}$. This means $\vec{w}$ = k $\vec{u}$

If $\vec{w}$ = k $\vec{u}$ and the two vectors have a common point, the points that make vectors $\vec{w}$ and $\vec{u}$ are collinear, or lie on the same line.

17.16 TRANSLATION VECTOR

Image vector $\vec{I}$ = Translation vector $\vec{T}$ + Object vector $\vec{O}$ i.e $\vec{I} = \vec{T} + \vec{O}$

TRANSLATION: Every point is moved the same amount in the same direction.

Object and Image are directly congruent (one can be fitted exactly on top of another without turning it over.)

The lines from object points to image points are all parallel.

17.17 POSITION VECTOR

If $\overrightarrow{OA} = \binom{a}{b}$, then $\overrightarrow{OA}$ is a position vector and the coordinates of A is A (a, b). The position vector is a vector with initial point as (0, 0).

17.18 MID-POINT OF A VECTOR

If $\vec{u} = \binom{a}{b}$ and $\vec{v} = \binom{c}{d}$, given that $\vec{u}$ and $\vec{v}$ are position vectors, the mid- point of the of the line $\overline{uv}$ is given by $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$

17.20 VECTORS II

REM: COORDINATES IN TWO AND THREE DIMENSIONS

Coordinates of points in two dimensions are of the form (x, y)

Coordinates of points in three dimensions are of the form (x, y, z)

17.21 COLUMN AND POSITION VECTORS IN THREE DIMENSION

The position vector of the point A(x, y) in two dimensions is $\binom{x}{y}$. In 3 – D, the position vector of the point B(x, y, z) is $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

The column vectors in three dimensions will always take the form $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ where a, b, and c are real numbers.

17.22 COLUMN VECTORS IN TERMS OF i,j and k

Vectors in three dimensions are handled the same way as vectors in two dimensions. The unit vectors used in 2 –D are **i** and **j**. such the position vector $\binom{x}{y}$ becomes $(x\boldsymbol{i} + y\boldsymbol{j})$ where i and j are unit vectors. In 3 – D the unit vectors are **i**, **j** and **k**. The vector $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ becomes $(x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k})$

Vectors in 3-D are added and subtracted the same way as in 2 – D. Multiplication by a scalar is done the same way.

17.23 MAGNITUDE OF A VECTOR IN THREE DIMENSIONS

The magnitude or size of the vector $(a\boldsymbol{i} + b\boldsymbol{j} + c\boldsymbol{k})$ is given by $\sqrt{a^2 + b^2 + c^2}$.

17.24 PARALLEL VECTORS AND COLLINEARITY

Two vectors are parallel if one is a scalar multiple of the other. If the vectors have a common point, the points involved are collinear. If PQ = kQR, PQ is parallel to QR. Because Q is common, P,Q and R are collinear.

17.25 THE RATIO THEOREM

- If points P and Q have positions vectors p and q, the point R divides the line PQ in the ratio m : n if and only if the position vector
  OR = $\frac{n}{m+n}\mathbf{p} + \frac{m}{m+n}\mathbf{q}$
- If points P and Q have positions vectors p and q, and the point R divides the line PQ in the ratio m :- n if and only if the position vector
  OR = $\frac{-n}{m-n}\mathbf{p} + \frac{m}{m-n}\mathbf{q}$.

18.00 TRANSFORMATIONS

18.10 ROTATION

18.11 PROPERTIES OF ROTATION

Rotation is described by the centre of rotation, angle of rotation and the direction of rotation. Anticlockwise rotation is positive rotation and Clockwise rotation is negative rotation.

To rotate an object about a centre through a given angle, the line joining each object point to the centre of rotation is rotated through the angle of rotation to the image point.

CONGRUENCE AND ROTATION

For rotation the image and object are directly congruent.

18.12 CENTRE OF ROTATION

To find the centre of rotation, join two matching points on the object and image. Draw the perpendicular bisector of this line. Repeat this for another pair. The two perpendicular lines meet at the centre of rotation.

18.13 ANGLE OF ROTATION

To find the angle of rotation, if the centre of rotation is known as above, the angle is between two lines joining an object point and its image point individually to the centre of rotation. A protractor is used to measure the angle of rotation.

18.14 ROTATIONAL SYMETRY

Rotational symmetry is the number of times an object or shape fits exactly into itself as it completes a rotation of 360^o about a point or its axis of symmetry.

18.20 REFLECTION AND CONGRUENCE

Reflection is described by the mirror line.

Object and image are oppositely (or indirectly) congruent (one can be fitted exactly on top of the other after being flipped over – turned over)

The line joining a point to its image is perpendicular to the mirror line. It is bisected by the mirror line. The perpendicular bisector of a line joining two matching points on the object and its image is the mirror line.

N.B: SYMMETRY

1. Lines of symmetry divide a plane shape into two matching halves.

2. Planes of symmetry divide a solid into two matching halves.

18.30 SIMILARITY AND ENLARGEMENT

18.31 SIMILARITY

PROPERTIESB OF SIMILAR FIGURES

Similar figures have the ratio of corresponding sides as equal, and the corresponding angles are equal.

18.32 ENLARGEMENT

CONSTRUCTION AND ENLARGEMENT IN THE CARTESIAN PLANE

To enlarge and object by a stated factor, first draw a straight line from the edge to the centre of enlargement, measure this distance. Multiply this measured length by the scale factor, to get the new length for the image point. With the line from the centre of enlargement through the object point produced, use the new length to locate the image point on it. Repeat this for all the other edges.

Linear Scale factor = image length / matching object length.

If the scale factor is positive, the image and object are on the same side of the centre of enlargement.

If the scale factor is negative, the image and object are on opposite sides of the centre of enlargement.

18.33 LINEAR, AREA AND VOLUME SCALE FACTORS AND THEIR RELATION

$A.s.f = (l.s.f)^2$ and $V.s.f = (l.s.f)^3$. To find corresponding $A.s.f$ from $V.s.f$ first find the l.s.f. Corresponding sides can be identified only from corresponding angles that have been matched. It's advisable to sketch out the figures involved separately to sort out questions on similarity and enlargement

19.00 MATRICES

A matrix is ordered as number of ROWS to number of COLUMNS e.g a mxn matrix has m rows by n columns.

A diagonal matrix has all diagonal elements equal and all other elements are zeros. A diagonal matrix is also called a SCALAR matrix. A square matrix has order nxn.

Two matrices A and B are equal, A = B, if and only if they have the same order and each element in A is equal to the corresponding element in B.

19.01 MATRIX COMPATIBILITY IN ADDITION AND SUBTRACTION

ADDITION AND SUBTRACTION OF MATRCES: If A and B are two matrices of the same order then,

The sum of A and B, A + B, is a matrix whose elements are the sum of the corresponding elements of A and B.

The differences of A and B, A – B, is a matrix whose elements are obtained by subtracting elements of B from the corresponding elements of A.

19.02 MATRIX COMPATIBILITY IN MULTIPLICATION

MULTIPLICATION OF A MATRIX BY A SCALAR k:

k being any real number, k**A** is the matrix obtained by multiplying each element of matrix **A** by k. If k = - 1, we get – **A**.

MULTIPLICATION OF MATRICES

Any matrix of order mxn and nxp ca be multiplied to give matrix of order mxp.

Consider $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax+by \\ cx+dy \end{pmatrix}$, to multiply a matrix A by a matrix B, take matrix A and multiply by each column in B. Then put the columns together to give the resulting matrix.

IDENTITY MATRIX

An IDENTITY matrix is a Diagonal matrix of order nxn whose diagonal elements are all equal to 1.

19.03 DETERMINANT OF A 2X2 MATRIX

A = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then the determinant, det (A) = $|\boldsymbol{A}|$ = $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ = $(ad - cb)$

19.04 INVERSE OF A 2X2 MATRIX

If **A** = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then the Inverse of **A** = $\boldsymbol{A}^{-1}$ = $\frac{1}{det\ \boldsymbol{A}}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

SINGULAR MATRIX

If the determinant is zero, the matrix has no inverse. The matrix is said to be singular matrix.

19.05 SOLUTION OF SIMULTANEOUS EQUATIONS BY MATRIX METHOD

Consider the set of equations $\begin{pmatrix} ax+by=e \\ cx+dy=f \end{pmatrix}$, set coefficient matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and find its inverse matrix which is $\frac{1}{ad-cb}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$, the values of x and y are gotten by:

- $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{ad-cb}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}\begin{pmatrix} e \\ f \end{pmatrix}$

20.00 MATICES AND TRANSFORMATIONS

20.01 IDENTIFICATION OF A TRANSFORMATION MATRIX

1. $\bar{T} + \bar{O} = \bar{I}$ **,** Translation vector + Object vector = Image vector.

2. $\tilde{T} \times \breve{O} = \breve{I}$**,** Transformation matrix × Object matrix = Image matrix.

3. The unit matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ which is a square matrix, where **I (1, 0) and J (0, 1),** on the Cartesian plane can be used to find the transformation matrices for:

(a) Reflection in the line: y=x, y=-x, x=0, y=0;

(b) Rotation about the origin (0,0) through 90^o, - 90^o, 180^o and -180^o.

The coordinates of the Image I' and J' give the transformation matrix.

4. If the Image and Object points are known using statement in no. 2, simultaneous equations result that can be solved to give the transformation matrix**.**

$\begin{pmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{pmatrix}$ **x** $\breve{O} = \breve{I}$**.** Equate corresponding elements after multiplying and solve resulting simultaneous equations**.**

20.02 SUCCESSIVE TRANSFORMATION

Successive transformations: If $\breve{X}, \breve{Y}$ and $\tilde{Z}$ are transformations then $\breve{X}\breve{Y}\tilde{Z}$ means perform $\tilde{Z}$ first, followed by $\breve{Y}$ and lastly $\breve{X}$. The single transformation $\tilde{P} = \breve{X}.\breve{Y}.\tilde{Z}$ will represent the three transformations.

SINGLE MATRIX OF TRANSFORMATION FOR SUCCESSIVE TRANSFORMATIONS

The single transformation $\tilde{P} = \breve{X}.\breve{Y}.\tilde{Z}$ will represent the three transformations, starting with **Z**, **Y** and then **X**.

20.03 INVERSE OF A TRANSFORMATION MATRIX

The inverse transformation matrix will always take the Image back to the Object. $\widetilde{A^{-1}}\ x\,\tilde{I} = \tilde{O}$

20.04 AREA SCALE FACTOR AND THE DETERMINANT OF A MATRIX OF TRANSFORMATION

The area scale factor, A. s. f is numerically equal to the determinant of the transformation matrix.

20.05 AREAS OF TRIANGLES AND QUADRILATERALS GIVEN THEIR COORDINATES

If the coordinates of a triangle or quadrilateral are known, their areas can be found as follows:

a) Triangles: A = $\frac{1}{2}\begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix}$

b) Quadrilateral:

A = $\frac{1}{2}[(x_1 - x_2)(y_1 + y_2) + (x_2 - x_3)(y_2 + y_3) + (x_3 - x_4)(y_3 + y_4) + (x_4 - x_1)(y_4 + y_1)]$

20.06 STRETCH TRANSFORMATION AND ITS MATRIX

All points not on the invariant line move in a direction perpendicular to the invariant line.

A stretch matrix with x – axis invariant and scale factor k is $\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$

A stretch matrix with y – axis invariant and scale factor k is $\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$

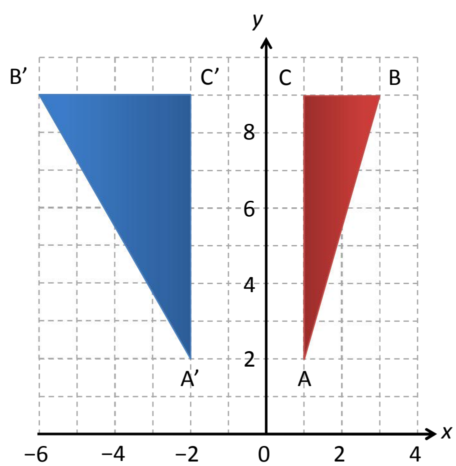
20.07 THE SCALE FACTOR k

$$k = \frac{distance\ from\ invariant\ line\ to\ the\ image\ point}{perpendicular\ distance\ from\ invariant\ line\ to\ object\ point} = \frac{image\ length}{corresponding\ object\ length}$$

If k is negative, the stretch occurs in the opposite direction.

The factor k can be used to locate the invariant line. k is the factor that multiplies the perpendicular distance of the object point from the invariant line to give the perpendicular distance of the corresponding image point from the invariant line.

If the scale factor is negative then the stretch is in the opposite direction.



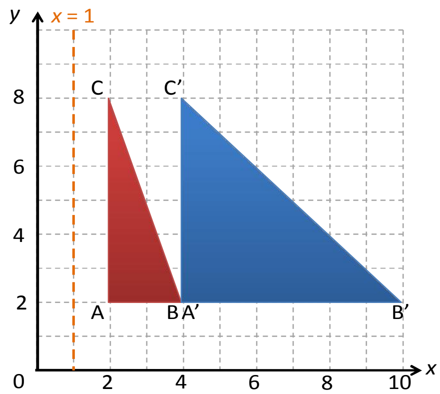
B′C′ = 2 × BC and it has been stretched in the opposite direction.

So the stretch factor is −2.

The perpendicular distance of each point from the *y* axis has doubled.

So the invariant line is the *y* axis.

The following diagram shows a stretch where the invariant line is not the *x* or *y* axis.



A'B' = 3 × AB

So the stretch factor is 3.

The perpendicular distance of each point from the line *x* = 1 has trebled.

So the invariant line is *x* = 1.

20.08 SHEAR TRANSFORMATION AND ITS MATRIX

Points on the invariant line do not move.

All the points not on the invariant line move parallel to the invariant line – fixed line.

A shear with x – axis invariant has matrix $\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$

A shear with y – axis invariant has matrix $\begin{pmatrix} 1 & 0 \\ k & 1 \end{pmatrix}$

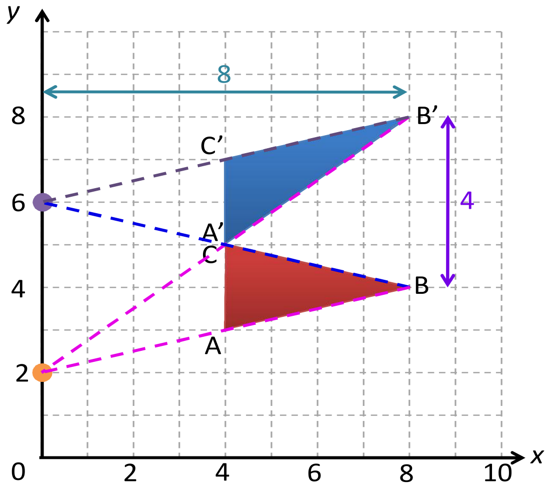
20.09 THE SHEAR FACTOR k,

$$k = \frac{\textit{distance moved by a point}}{\textit{perpendicular distance of point from the invariant line.}}$$

If k is negative, shear occurs in opposite directions.

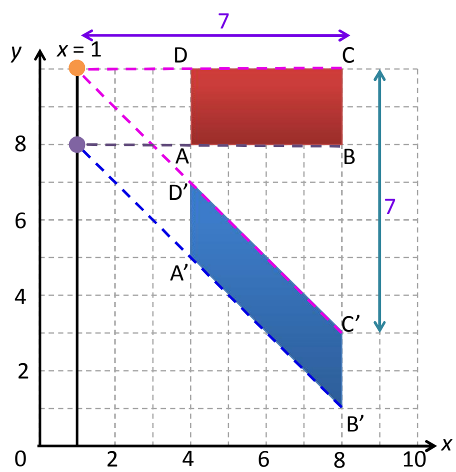
If the image A'B'C'D' is show and its object ABCD, the invariant line can be located by: (a) Join suitably BA produced to meet B'A' produced at P. (b) Repeat this for another pair to meet at Q. The line PQ produced is the invariant line**.**

6 Describe fully the single transformation that takes ABC onto A'B'C'.



- shear
- invariant line is the *y* axis
- shear factor is $\frac{4}{8} = \frac{1}{2}$

8 Describe fully the single transformation that takes ABCD onto A'B'C'D'.



- shear

note: this is a negative shear

- invariant line is $x = 1$
- shear factor is $-\frac{7}{7} = -1$

21.00 NAVIGATIOIN

21.10 LONGITUDES AND LATTITUDES

21.11 DISTANCES BETWEEN TWO POINTS ALONG GREAT CIRCLES IN NAUTICAL MILES AND IN KILOMETRES

1. If an arc of a great circle subtends an angle of α at the centre of the earth, the arc's length is given by $(l) = (60\alpha)$nm OR $(l) = \frac{\alpha}{360} \times (2\pi R)$ km.

2. The shortest distance between two points on the earth's surface is the distance along a great circle.

21.12 DISTANCE ALONG A SMALL CIRCLE OF LATITUDE I NAUTICAL MILES AND KILOMETRES

If the angle at the centre is α at the centre of a latitude angle ϴ, then the length of the arc is given by $(l) =$ $(60\alpha\ cos\theta)nm$ OR $(l) = \frac{\alpha}{360} \times (2\pi R)\ (cos\theta)$ km. ϴ is the latitude angle and $\alpha\ is\ \ longitude$ difference for the two points on the arc.

21.13 TIME AND LONGITUDES

For every 1^o change in longitude there is a corresponding change in time of 4 minutes. Actually, 15^o apart means 1 hour difference in time.

21.14 SPEEDS IN KNOTS AND KILOMETERS PER HOUR

Speed = nautical miles/time in hours. = speed in knots. 1 knot = 1nm/hr.

22 .00 AREA APPROXIMZATION

22.01 AREA BY COUNTING TECHNIQUE

1. Count all the complete squares inside the figure.

2. Count all the partially complete squares, and divide by 2.

3. Add answers 1. and 2 to get the total area enclosed.

4. Use the scale provided to get the actual area.

22.02 TRAPEZIUM RULE AND AREA

The trapezium rule for estimating the area under a curve and the x-axis:

$\int_a^b y dx = \frac{h}{2}[y_0 + 2(y_1 + y_2 + y_3 + y_4 + y_5 - - - - + y_{n-1}) + y_n]$, where n is number of strips and the width h of each strip is given by the $(h) = \left(\frac{b-a}{n}\right)$.

The number of ordinates is one more than the number of strips.

This rule simply gives an estimate of the area.

22.03 MID-ORDINATE RULE AND AREA

The Mid-ordinate rule for estimating the area under a curve and the x – axis**:**

$\int_a^b y dx = (h)\,[(y_1 + y_2 + y_3 + y_4 + y_5 - - - - + y_n)]$, where **n** is the number of strips and the width h of each strip is given by $(h) = \left(\frac{b-a}{n}\right)$.

The first value of x of the first rectangle is given by $x_1 = (a) + \frac{h}{2}$

Subsequent values of x are consecutively h apart**.**

The number of ordinates is equal to the number of strips

23.00 ELEMENTARY CALCULUS

23.10 DIFFERENTIATION

23.11 GRADIENT OF A CURVE (y = ax^n) AT A POINT

1. The gradient of a curve y = f(x) at the point x = a is the gradient of the tangent to the curve at x = a.

2. $\frac{dy}{dx}$ is the derivative of y with respect to x. It represents the rate of change of y with respect to x.

3. If y = f(x) then $\frac{dy}{dx}$ = $f'(x)$. 4. If y = x^n then $\frac{dy}{dx}$ = nx^{n-1} 5. If y = mx^n where m is a constant, $\frac{dy}{dx}$ =mnx^{n-1}

6. If y = u $\pm$ v, where u and v are functionss of x, then $\frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$

23.12 EQUATIONS OF TANGENTS AND NORMALS TO THE CURVE

1. The equation of a tangent to the curve y = f(x) at the point on the curve with coordinates (a, b) is y – b = $f'(a)$ $(x - a)$

2. The equation of the normal to the curve y = f(x) at the point on the curve with coordinates (a, b) is y – b =$\frac{1}{f'(a)}$ $(x - a)$

23.13 STATIONARY POINTS

1. A point on the curve y = f(x) for which $f'(x) = 0$ is called a turning point (or stationary point) of the curve. This can be a maximum, minimum or a point of inflexion.

2. If $f'(x) = 0$ and $f''(x) < 0$, the turning point is a maximum.

3. If $f'(x) = 0$ and $f''(x) > 0$, the turning point is a minimum

4. If $f'(x) = 0$ and $f''(x) = 0$, and $f'''(x) \neq 0$ the turning point is a point of inflexion.

23.14 CURVE SKETCHING

Curve sketching can be done using the stationary points, the x- and y- intercepts.

23.15 APPLICATIONS TO VELOCITY AND ACCELERATION

Distance expression is either $s(t)$ *or* $x(t)$, the position of the object relative to a fixed point at time t.

Velocity expression $v(t) = \frac{ds}{dt}$ *or* $\frac{dx}{dt}$, the instantaneous velocity of the object.

Acceleration expression $a(t) = \frac{dv}{dt}$, instantaneous acceleration of the object.

Initial means t = 0 and at rest means v = 0. If initial conditions are given or known, the actual values of the expressions can be determined.

23.16 MAXIMA AND MINIMA

Maxima and minima are applied to lengths, area and volumes. Conditions could be provided that can lead to the formation of a function. Such a function could require optimization that can only be done via differentiation.

23.20 INTEGRATION

REVERSE OF DIFFERENTIATION

Given $\frac{dy}{dx} = f'(x)$ as an indefinite integral, y = f(x) + C, where C is an arbitrary constant.

23.21 INDEFINITE INTEGRALS

1. $\int ax^n \, dx = \frac{a}{n+1} x^{n+1}$ + C, where n$\neq -1$

2. $\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$ where $n \neq -1$

3 . $\int (u+v) \, dx = \int u \, dx + \int v \, dx$ + C where u and v are functions of x and C an arbitrary constant.

23.22 DEFINATE INTEGRALS

$\int_a^b f'(x) \, dx = [f(x)]_a^b$ = f(b) - f(a)

23.23 AREA UNDER A CURVE BY INTEGRATION

1. The area of the region bounded by the curve y = f(x), the coordinates x = (a) and x = (b) and the x – axis can be found be evaluating the definite integral $\int_a^b f(x) \, dx$.

2. The area enclosed by graphs $f_1(x)$ and $f_2(x)$ is given by $\int_a^b f_2(x) - f_1(x) \; dx$, where a and b are the x – coordinates of the points of intersection of the two graphs.

N.B. Note that it's good to have a sketch of the curve(s) before applying integration to get the area. Area below the x-axis will always be negative. If in part of the area is below the x-axis, sections 5. and 6. above should be done in steps. Each area should be evaluated separately. The negative sign for the area below the x-axis is neglected.

23.24 APPILICATIONS TO VELOCITY AND ACCELERATION

Velocity expression $v(t) = \int a(t) \, dt$, the instantaneous velocity of the object at a point.

Distance expression $s(t) or \; x(t) = \int v(t) dt$, the position of an object relative to a fixed point at time t.

If the limits are given, know or found the actual values for the expressions can be found.