NATIONAL BUSINESS AND TECHNICAL EXAMINATIONS BOARD
NBC/NTC EXAMINATION
MATHEMATICS

1(a) Solve for x in $8^{3x} \times 8^{-1} = 32$

(b) Simplifying without using tables, $\frac{\log 27}{\log 3}$

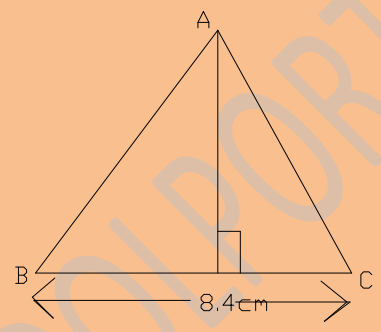
Solution

(a) $2^{3(3x)} \times 2^{3(-1)} = 2^5$
$\Rightarrow 3(3x) - 3 = 5$
$9x - 3 = 5$
$\therefore x = 8/9$

(b) $\frac{\log 27}{\log 3} = \frac{\log 3^3}{\log 3} = \frac{3\log 3}{\log 3} = 3$

2(a) The 6th term of a G.P is 1215. If the common ratio is 3; find its 3rd term.

(b) ABC is a triangle with BC = 8.4cm, $\angle ADC = 90^o$ and area $40.16cm^2$. Find /AD/.



Solution

(a) $T_6 = ar^{n-1} \quad \Rightarrow \quad a(3)^5 = 1215$

$a = \frac{1215}{243} = 5$

$\therefore 3^{rd}$ term $= 5 \times 3^2 = 45$

(b) Area of a triangle = ½ x 8.4 x /AD/
$= 40.16cm^2$

$\therefore /AD/ = \frac{40.16 \times 2}{8.4 \times 1}$

$= 9.56cm$

3(a) Simplify $\frac{0.0054 \times 8.19}{0.000243}$, leaving your answer in standard form.

(b) A length of 8.85m is increased to 9.37m. Calculate the increase.

Solution

(a) $\frac{54 \times 10^{-3} \times 819 \times 10^{-2}}{243 \times 10^{-5}}$

$= 1.82 \times 10^{2}$

or $\frac{0.054 \times 819}{0.00243} = \frac{0.44226}{0.00243} = 182$

$= 1.82 \text{ X } 10^{2}$

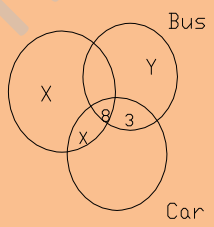
(b) increase in length = (9.37 – 8.85)m

= 0.52m

percentage increase = $\frac{0.52}{8.85} \times 100$

= 5.876% = 5.88% approx.

4. 65 of the workers in a certain company in Lagos were interviewed about the means of transportation to work on a particular day. Each of them used one or more of the means shown on the Venn diagram below.



Given that 37 workers used Bike and 20 used Bus, find

(a) x

(b) the number of workers who used cars only

Solution

4(a) $x + x + 5 + 8 = 37$

$2x = 24$

$\therefore$ x =12

(b) y = 20 – (5+8+3) = 4

n (Bike $\cup$ Bus) = 12+12+5+8+3+4 = 44

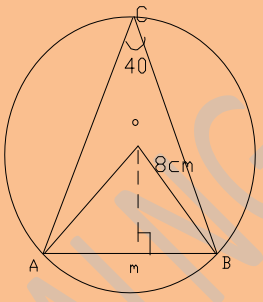
n (cars only) = 65- 44

= 21

5. The centre of the circle ABC is O. If its radius is 8cm and < ACB = 40^o, Calculate the length of the

(a)) Chord AB

(b) Perpendicular OM



Solution

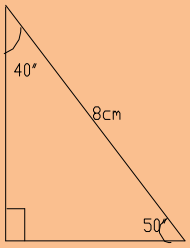
< AOB = 2<ABC = 2 x 40^o = 80^o

< BOM = ½ of 80^o = 40^o

Considering triangle OMB,

/MB/ = 8 sin 40^o

or 8 Cos 50^O = 5.142cm

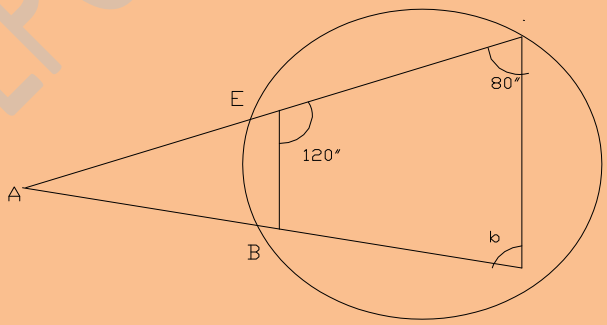


Length of the chord AB = 2/MB/ = 2 x 5.142
= 10.28cm = 10.3cm approx

(b) /OM/= 8 Cos 40^o or 8Sin50^o
=6.128cm = 6.13cm = 6.1cm

ALITER: Using Pythagoras' rule
OM = $\sqrt{(OB)^2 - (MB)^2}$ = 6.1cm

6(a) Find the value of a and b in the figure below



(b) Five years ago, a father was twice as old as his son. In 4 years' time, the sum of their ages will be 78. Find their present ages.

Solution(a) b = $180^o - 120^o = 60^o$ (opposite angles in cyclic quad are supplementary)
Considering ΔACD,

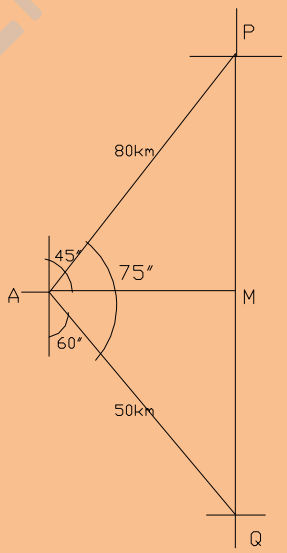
$a + b + 80^o = 180^o$ ($\angle$s in a Δ)
$\therefore a = 180^o - 80^o - 60^o = 40^o$

(b) Let the present ages be son, x yrs, father y yrs,
then 5 years ago, we have
$y - 5 = 2(x-5)$
$\Rightarrow 2x - y = 5$ _____(1)
in 4 years' time, we have
$(x+4) + (y+4) = 78$
$\Rightarrow x+y = 70$ _____(2)
From (1) and (2), we have, $x = 25$ and $y = 45$
$\therefore$ their present ages are son = 25 yrs, father = 45 years

(a) ALITER
5 year ago if son is y year's old father was 2y years old. In 4 years time, son will be (y+5+4) yrs
father = (2y+5+4) yrs
which gives $y + 9 + 2y + 8 = 78$; $y = 20$
the present ages are $y + 5 = 25$yrs and $2y + 5 = 45$yrs for the son and father respectively.

7. The bearings of points P and Q from 045^o and 120^o respectively. If the distance AP is 80km and AQ is 50km, calculate the:
(a)) distance between P and Q to 3 significant figures
(b) bearing of Q from P, to the nearest degree.
(c)) how far east of A is Q?

Solution
(a)



Correct diagram with at least three of 50km, 80km, 45^o, 60^o or 120^o shown <PAQ = 75^o

$(PQ)^2 = 80^2 + 50^2 - (50)\ Cos75^o = 6829.6$

$\therefore PQ = \sqrt{6829.6} = 82.6km$

Solution

7(b) $Sin < APQ = \frac{50 \times sin\ 75^o}{82.6} = 0.5847$

$< APQ = sin^{-1}\ 0.5847 = 35.78^o$

$< QPN = 45^o - 35.78^o = 9.22^o$

The bearing of Q from P = $180^0 + 9.22^0$

$= 189^o$ (to the nearest degree).

$\therefore < QAM = 30^o$

(c) A is 50 x Cos 30^o = 43.3km east of Q

8(a) The table below shows the scores of a group of 40 students in a test.

Score (x)	1	2	3	4	5	6	7	8	9	10
Frequency (f)	3	4	5	7	8	6	3	2	1	1

Find the (i) mode, (ii) median and (iii) mean

(b) The 2^{nd} and 4^{th} terms of a G.P. are 10 and 40 respectively. Find the

(i) common ratio

(ii) first term

(iii) 8^{th} term of the series

Solution

(a) (i) mode = 5

(ii) median= $\frac{5 + 5}{2} = 5$

(iii) $\Sigma fx = 3 + 8 + 15 + 28 + 40 + 36 + 21 + 16 + 9 + 10$

$= 186$

mean = $\frac{186}{40} = 4.65$

(b) $ar^{2-1} = 10 = ar = 10$

$ar^{4-1} = 40 = ar^3 = 40$

$\Rightarrow r^2 = \frac{40}{10} = 4$

(i) $\therefore r = \pm 2$, r = 2 or -2

(ii) Hence $2a = \pm 10 \Rightarrow a = \pm 5$

(iii) $T_8 = \pm 5 \times 2^7 = 640$

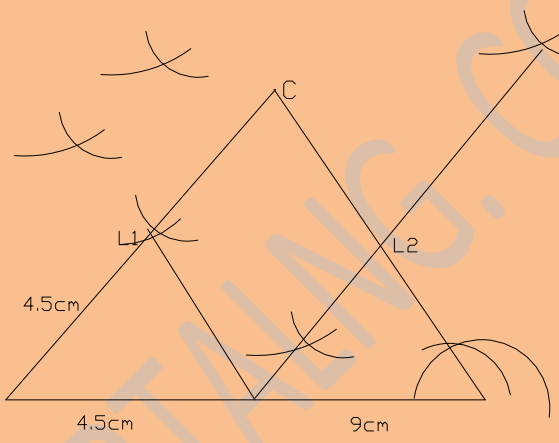
9 Using a ruler and a pair of compasses only construct:

(a) A triangle ABC such that /AB/=9cm,
$\angle ABC = 60^o$ and $\angle ACB = 45^{o.}$

(b) (i) Construct the locus l_1 of points 4.5cm from A.
(ii) Construct the locus l_2 of points equidistant from B and C to intersect l_1 at x_1 and x_2 measure /x_1x_2/.

Solution

(a) Drawing a side 9cm long constructing angle 60^o, angle 45^o measuring angle BAC = 76^o completing the triangle ABC.



(b) (i) Constructing l_1 4.5cm from A
(ii) Constructing l_2 of points equidistant from B and C to intersect l_1 at x_1 and x_2
measuring /x_1x_2/ = 8.5cm; ($\pm$ 0.1cm) or its equivalent.

10(a) A bucket is 28cm in diameter at the top, 18cm in diameter at the bottom and 20cm deep. Find the capacity, in litres, of the bucket (take $\pi = 3.142$)

(b) The hypotenuse of a right angled triangle is 17cm and one of the angles is $43^{o,}$ find the
(i) third angle
(ii) side opposite the smallest angle.

Solution

(a) Let the height of the smaller cone be h cm then, we have $\frac{h}{20+h} = \frac{9}{14}$

$\Rightarrow$ 14h = 180+9h
$\therefore$ h = 36

Volume of the small cone = $\frac{1}{3}$ x 3.142 x 9^2 x 36

= 3054.02cm^3

Volume of the big cone = $\frac{1}{3}$ x 3.142 x 14^2 x 56

= 11495.53cm^3

Volume of the bucket in litres = 11495.53 – 3054.02

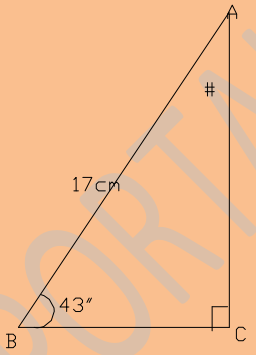
= 8441.51cm^3

Capacity of the bucket in litres = 8.44 litres or 8.4 litres.

We can also get the volume if we use $\frac{\pi}{3}(r^2H - r^2h)$

Substituting for R, H, r and h, we get 8.44 litres

3^{rd} angle Ø = 180^o- (90^o+43^o)=47'



AC is opposite the smallest angle

Hence AC = 17 x Sin 43^o

or AC = 17 x Cos 47^0

= 11.594cm

or = 11.59cm

11(a) The sum to nth term of an AP is given by

$S = \frac{n}{2}[a + (n-1)d]$, where a = first term

d = common difference

(i) make d the subject of the formula

(ii) Hence calculate the common difference of an AP whose sum is 338, n = 13 and a = 5.

(b) The angles of a polygon are $(x-10)^0$, x^o, x^o, $(x+20)^o$ and $(x+30)^o$. Find the value of x^0.

Solution

$S = \frac{n}{2}(a+(n-1)d)$

2

Removing the fraction and brackets to get $2s = na + n^2d - nd$

Isolating d, we get

$d(n^2 - n) = 2s - na$

$\therefore d = \frac{2s - na}{n^2 - n}$

(ii) $d = \frac{2s - na}{n^2 - n} = \frac{2(338) - 13(5)}{13^2 - 13}$

$= \frac{611}{156} = 3.92$

(b) The polygon has 5 sides

sum of interior angles = $3 \times 180^o = 540^o$

$(x-10)^o + x^o + x^o + (x + 20)^o + (x + 30)^o = 540^o$

$5x + 40^o = 540^o$

$5x = 500^o$

$\therefore x = 100^o$

12(a) An article costing ₦32.50 is sold for a gain of 13½. Find the selling price.

(b) Find the simple interest on ₦4500.00 in 2½ years at 4% per annum.

(c) A businessman borrowed ₦200,000 from a bank for 3 years at 5% compound interest.

(i) Calculate the interest on the loan at the end of the period.

(ii) If he repays ₦230,000 at the end of the 3 years, how much does he still owe?

Solution

(a) Cost price of the article : 100% = ₦32.50

Selling price of the article 113½ % = $₦\frac{113.5}{100} \times 32.50$

= ₦36.89

(b) $S.I = \frac{PTR}{100} = \frac{4500 \times 5 \times 4}{100 \times 2}$

= ₦450.00

(c) Interest at the end of 1st year = $₦\frac{200,000 \times 1 \times 5}{100}$

= ₦10,000.00

Interest at the end of 2nd year = $\frac{210,000 \times 1 \times 5}{100}$

= 10,500.00

Interest at the end of 3rd year = $₦\frac{220,500 \times 1 \times 5}{100}$

= ₦11,025.00

(i) Total interest owed at the end of 3rd year

$= ₦200{,}000\left(1+\frac{5}{100}\right)^3 = ₦231{,}525.00$

Total interest = ₦ (231,525 – 200,000) = ₦31,525.00

ALITER

(i) Total interest = ₦(10,000 + 10,500 + 11,025) = ₦31,525.00

(ii) Amount still owed = (231,525 – 230, 000)

= ₦1,525.00

13(a) A trader allows a discount of $33\frac{1}{3}$ % on his marked prices. What should be the marked prices of article he wishes to receive ₦500.00?

(b) The prices of kerosene per litre on the first week of each of the 12 months of the year are as given in the table below.

Month	Jan.	Feb.	March	April	May	June	July	Aug.	Sept	Oct	Nov	Dec.
Price	18	21	25	30	40	52	48	50	55	43	26	18

Find the three month moving averages for the period.

Solution

(a) Selling price less discount: 66 2/3% = ₦500

Marked price: 100% = ₦$\frac{100}{66\ 2/3}$ x 500

= 749.96 = ₦750 approx

(b) Moving averages: $\frac{18 + 21 + 25}{3} = 21.33$

$\frac{21+25+30}{3} = 25.33$, $\frac{25+30+40}{3} = 31.67$

$\frac{30+40+52}{3} = 40.67$, $\frac{40+52+48}{3} = 46.67$

$\frac{52+48+50}{3} = 50.00$, $\frac{48+50+55}{3} = 51$

$\frac{50+55+43}{3} = 49.33$, $\frac{55+43+26}{3} = 41.33$

$\frac{43+26+18}{3} = 29.00$

14(a) A man's salary is 298,886.40 per annum. Before receiving his salary, the employer makes the following deductions of the salary less personal allowance

Income Tax ……1%,
Federal Housing Scheme …2½%, and
union dues ……...2%
If his annual personal allowances is ₦108,110.40
Calculate:
(i) his monthly income tax
(ii) the net monthly take home pay.

(b) A bankrupt's assets realize ₦5000.00 and his liabilities are ₦8000.00
(i) What dividend will he pay?
(ii) How much will be paid to a creditor for ₦600.00?

Solution

(a) Salary – Personal allowances = ₦ (298,886.40 – 108,110.40)
= ₦190,776.00

(i) Monthly income tax = $\frac{190,776}{100} \times \frac{1}{12}$
= ₦158.98

(ii) Gross monthly salary = $\frac{₦298,886.4}{12}$
= ₦24,907.20

monthly deductions: FHS: 2% of ₦24,907.20
=₦ 622.68

monthly union due = 2% of ₦24,907.20
= ₦498.14

monthly tax deduction: 1% of ₦24907.21
= ₦249.07

monthly total deductions = ₦ (622.68 + 498.14 + 249.07)
= ₦1369.89

Net monthly pay = (24,907.40 – 1369.89)
= ₦23,537.31

(b) (i) Dividend = $\frac{₦5,000}{₦8,000} \times 100k$
= 63k in ₦

(ii) To a creditor for ₦600 he pays $\frac{63}{100} \times ₦600$
= ₦378.00

15(a) Find the weighted mean of 15,20,25,30, if they are assigned weightings of 2,1,3,4 respectively.

(b) A man bought 23 crates of bottled drink at ₦310.00 per crate. There were 24 bottles per crate and each bottle was sold for ₦15. If two bottles per crate got broken during sales, calculate the following:

(i) cost price of the 23 crates
(ii) percentage profit per crate.

Solution

(a) weighted mean = $\frac{(2 \times 15) + (1 \times 20) + (3 \times 25) + (4 \times 30)}{2 + 1 + 3 + 4}$

$= \frac{275}{10} = 27.5$

(b) (i) Cost price of 23 crates = 23 x ₦310
= ₦7130.00
No of bottles sold in a crate = 22
Selling price of a crate = 22 x ₦15 = ₦310.00
Profit on a crate = ₦330.00 – ₦310.00
= ₦20

(ii) Percentage profit per crate
$= \frac{20}{310} \times 100$
= 6.45% = 6.5% approx.